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TIME-FREQUENCY METHODS FOR SIGNAL DETECTION WITH APPLICATION TO THE DETECTION OF KNOCK IN CAR ENGINES*

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ABSTRACT

We propose time-frequency (TF) formulations, efficient and stable TF design procedures, and an efficient TF implementation for quadratic detectors. Conventional and TF detectors are applied to knock detection and compared with regard to performance and required *a priori* knowledge. It is shown that TF detectors are advantageous under realistic conditions where robustness and stability are important.

1 INTRODUCTION

We consider the problem of discriminating between two signals $s_0(t), s_1(t)$ embedded in stationary white Gaussian noise $n(t)$ with power spectral density η [1, 2]. The hypotheses are $H_i : r(t) = r_i(t) = s_i(t) + n(t)$ $(i = 0, 1)$. The detector computes a quadratic test statistic

$$\Lambda(r) = \langle \mathbf{H}r, r\rangle = \int_t \int_{t'} h(t,t')\, r(t')\, r^*(t)\, dt\, dt',$$

i.e., the quadratic form of the observed signal $r(t)$ induced by a linear operator $\mathbf{H}$ with kernel $h(t,t')$. This is compared to a threshold to obtain the final decision. In the remainder of this introduction, we consider several quadratic test statistics that are suited to various models for $s_0(t)$, $s_1(t)$ and various levels of available *a priori* knowledge. In Section 2, we propose time-frequency (TF) formulations of these test statistics that yield efficient and stable TF designs of detectors. Whereas much of the existing work on TF detectors considered exact TF implementations of optimal test statistics [3]–[6], our TF detectors are approximations of the type introduced in [7, 8] that are valid for the practically important class of *underspread* nonstationary processes [9]–[11]. Section 3 discusses an efficient TF implementation of quadratic detectors. Finally, the detectors are applied to knock detection in Section 4.

1.1 Likelihood Ratio Detector

First, we model $s_0(t)$, $s_1(t)$ as Gaussian nonstationary random processes with known correlation operators[1] $\mathbf{R}_{s_0}$, $\mathbf{R}_{s_1}$. Here, the optimal (likelihood ratio) test statistic is [1, 2]

$$\Lambda_1(r) = \langle \mathbf{H}_1 r, r\rangle \quad \text{with } \mathbf{H}_1 = \mathbf{R}_0^{-1}(\mathbf{R}_1 - \mathbf{R}_0)\mathbf{R}_1^{-1}, \quad (1)$$

where $\mathbf{R}_i = \mathbf{R}_{s_i} + \eta\mathbf{I}$ $(i = 0, 1)$. For low SNR, i.e.[2] $\|\mathbf{R}_{s_i}\| \ll \eta$, the optimal test statistic Λ_1 can be approximated by the following "locally optimal" [1] test statistic that is induced by a much simpler operator:

$$\Lambda_2(r) = \langle \mathbf{H}_2 r, r\rangle \quad \text{with } \mathbf{H}_2 = \mathbf{R}_1 - \mathbf{R}_0 . \quad (2)$$

The $\mathbf{R}_i$ can be estimated from N_i observations $r_i^{(n)}(t)$ $(i = 0, 1;\ n = 1, 2, ..., N_i)$ by the sample correlation operators $\hat{\mathbf{R}}_i$ with kernels $\hat{r}_i(t,t') = \frac{1}{N_i}\sum_{n=1}^{N_i} r_i^{(n)}(t)\, r_i^{(n)*}(t')$.

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[1]For example, $\mathbf{R}_{s_0}$ is the linear operator with kernel $r_{s_0}(t,t') = \mathrm{E}\{s_0(t)\, s_0^*(t')\}$.

[2]Here, $\|\mathbf{H}\|$ denotes the operator norm [12].

1.2 Matched Subspace Detector

Next, we model $s_0(t)$, $s_1(t)$ as deterministic but unknown except for $s_i(t) \in \mathcal{S}$ with $\mathcal{S}$ a known p-dimensional linear signal subspace, and $\|s_0\|^2 \le E$ and $\|s_1\|^2 > E$ with E some known energy level. Here, an optimal test statistic is the *matched subspace detector* [2] given by the quadratic form induced by the orthogonal projection operator $\mathbf{P}_\mathcal{S}$ on $\mathcal{S}$, which equals the energy of $(\mathbf{P}_\mathcal{S} r)(t)$,

$$\Lambda_3(r) = \langle \mathbf{H}_3 r, r\rangle = \langle \mathbf{P}_\mathcal{S} r, r\rangle = \|\mathbf{P}_\mathcal{S} r\|^2 . \quad (3)$$

The p-dimensional subspace $\mathcal{S}$ can be estimated via *ML subspace identification* [2]. The estimated subspace is spanned by the p dominant eigenfunctions of the total sample correlation operator $\hat{\mathbf{R}} = \frac{1}{N_0+N_1}(N_0\hat{\mathbf{R}}_0 + N_1\hat{\mathbf{R}}_1)$.

1.3 Extended Matched Subspace Detector

As a model somewhere between the models of Subsections 1.1 and 1.2, we assume that $s_0(t)$, $s_1(t)$ are nonstationary random processes whose energies in K disjoint signal subspaces $\mathcal{S}_k$, $\langle \mathbf{P}_{\mathcal{S}_k} s_i, s_i\rangle = \|\mathbf{P}_{\mathcal{S}_k} s_i\|^2$, have known statistical properties. A reasonable detection strategy then is to use the subspace energies of the observed signal $r(t)$,

$$\xi_k \triangleq \langle \mathbf{P}_{\mathcal{S}_k} r, r\rangle = \|\mathbf{P}_{\mathcal{S}_k} r\|^2, \qquad k = 1, 2, \cdots, K,$$

as an intermediate statistic and devise an optimal (i.e., likelihood ratio) test based on the ξ_k. Specifically, if the $s_i(t)$ are Gaussian and the $\mathcal{S}_k$ have small dimensions so that $\mathbf{P}_{\mathcal{S}_k}\mathbf{R}_i\mathbf{P}_{\mathcal{S}_k}$ has only one dominant eigenvalue $\lambda_i^{(k)}$, then under hypothesis $\mathbf{H}_i$ the ξ_k are (approximately) independent and exponentially distributed with parameters $\theta_i^{(k)} = 1/\lambda_i^{(k)}$ [2]. Here, the likelihood ratio test statistic is

$$\Lambda_4(r) = \sum_{k=1}^{K} \left[\theta_0^{(k)} - \theta_1^{(k)}\right] \xi_k = \langle \mathbf{H}_4 r, r\rangle , \quad (4)$$

i.e., the quadratic form with operator

$$\mathbf{H}_4 = \sum_{k=1}^{K} \left[\theta_0^{(k)} - \theta_1^{(k)}\right] \mathbf{P}_{\mathcal{S}_k} .$$

This extends the matched subspace detector Λ_3 since for $\mathcal{S} = \bigoplus_{k=1}^K \mathcal{S}_k$ we obtain $\Lambda_3(r) = \sum_{k=1}^K \xi_k$.

2 TIME-FREQUENCY FORMULATION AND DESIGN OF DETECTORS

The detectors discussed above involve operator products, operator inverses, and signal spaces, which leads to computationally expensive design procedures. We shall now discuss efficient, stable, and intuitive formulations and designs of these detectors in the time-frequency (TF) domain.

2.1 Unified Time-Frequency Formulation

It is known [4, 6, 7] that all quadratic test statistics can be rewritten as inner products in the TF domain,

$$\Lambda(r) = \langle \mathbf{H}r, r\rangle = \langle \rho, W_r\rangle = \int_t\int_f \rho(t,f)\, W_r(t,f)\, dt\, df.$$

Here,

$$\rho(t,f) = L_{\mathbf{H}}(t,f) \triangleq \int_\tau h\Big(t+\frac{\tau}{2},\, t-\frac{\tau}{2}\Big)\, e^{-j2\pi f\tau} d\tau$$

is the *Weyl symbol* (WS) [13]–[15] of $\mathbf{H}$ and

$$W_r(t,f) \triangleq \int_\tau r\Big(t+\frac{\tau}{2}\Big)\, r^*\Big(t-\frac{\tau}{2}\Big)\, e^{-j2\pi f\tau} d\tau$$

is the *Wigner distribution* [16, 17] of $r(t)$. Thus, any quadratic test statistic can be interpreted as a weighted integral of $W_r(t,f)$. However, instead of using this exact TF formulation, here we shall typically consider approximations to the TF weight functions $\rho(t,f)$ that are obtained by replacing the WS of operator products by the products of the WSs of the individual operators. Such an approximation is justified for underspread processes [7]–[11].

2.2 TF Version of Likelihood Ratio Detector

For the optimal (likelihood ratio) detector for Gaussian processes, Λ_1 in (1), the TF weight function is given by

$$\rho_1(t,f) = L_{\mathbf{H}_1}(t,f) = L_{\mathbf{R}_0^{-1}(\mathbf{R}_1-\mathbf{R}_0)\mathbf{R}_1^{-1}}(t,f)\,.$$

For *jointly underspread* processes $s_0(t)$, $s_1(t)$ [9]–[11], this can be approximated by [7, 8]

$$\tilde{\rho}_1(t,f) \triangleq \frac{L_{\mathbf{R}_1}(t,f) - L_{\mathbf{R}_0}(t,f)}{L_{\mathbf{R}_0}(t,f) L_{\mathbf{R}_1}(t,f)} = \frac{\overline{W}_1(t,f) - \overline{W}_0(t,f)}{\overline{W}_0(t,f)\overline{W}_1(t,f)}\,, \tag{5}$$

where $\overline{W}_i(t,f) \triangleq L_{\mathbf{R}_i}(t,f)$ is the *Wigner-Ville spectrum* [18, 19] of $r_i(t)$. This yields the *TF test statistic*

$$\tilde{\Lambda}_1(r) \triangleq \langle \tilde{\rho}_1, W_r\rangle = \int_t\int_f \tilde{\rho}_1(t,f)\, W_r(t,f)\, dt\, df\,.$$

While $\tilde{\Lambda}_1$ is here expressed as a TF integral, it can be calculated more efficiently as the quadratic form $\tilde{\Lambda}_1(r) = \langle \widetilde{\mathbf{H}}_1 r, r\rangle$. Here, the operator $\widetilde{\mathbf{H}}_1$ is defined by $L_{\widetilde{\mathbf{H}}_1}(t,f) = \tilde{\rho}_1(t,f)$, from which the kernel of $\widetilde{\mathbf{H}}_1$ is obtained as

$$\tilde{h}_1(t,t') = \int_f \tilde{\rho}_1\left(\frac{t+t'}{2}, f\right) e^{j2\pi(t-t')f} df\,.$$

Furthermore, an efficient TF implementation of $\tilde{\Lambda}_1$ will be discussed in Section 3.

For jointly underspread $s_i(t)$, $\tilde{\rho}_1(t,f) = L_{\widetilde{\mathbf{H}}_1}(t,f)$ approximates $\rho_1(t,f) = L_{\mathbf{H}_1}(t,f)$. Hence, $\widetilde{\mathbf{H}}_1 \approx \mathbf{H}_1$ which implies that $\tilde{\Lambda}_1$ approximates the optimal test statistic Λ_1. The TF test statistic $\tilde{\Lambda}_1$ has two advantages over Λ_1:

- While both Λ_1 and $\tilde{\Lambda}_1$ require complete knowledge of the processes' second-order statistics, for the TF detector $\tilde{\Lambda}_1$ this knowledge is formulated in the much more intuitive and physically meaningful TF domain. The definition of $\widetilde{\mathbf{H}}_1$ is more transparent than the abstract operator expression defining $\mathbf{H}_1$.
- Computing $\widetilde{\mathbf{H}}_1$ requires only products and divisions of functions and thus is much less expensive than computing $\mathbf{H}_1$ which requires products and inverses of operators. It is also more stable since in the case of estimation errors the divisions in (5) can be regularized much more easily than the operator inversions in (1).

For the locally optimum test statistic Λ_2 in (2), the TF weight function becomes

$$\rho_2(t,f) = L_{\mathbf{H}_2}(t,f) = \overline{W}_1(t,f) - \overline{W}_0(t,f)\,,$$

which yields the following TF formulation of Λ_2,

$$\Lambda_2(r) = \langle \overline{W}_1 - \overline{W}_0, W_r\rangle\,. \tag{6}$$

This TF formulation is exact, i.e., not based on an underspread approximation. It extends a frequency-domain formulation of Λ_2 valid in the stationary case [1]. For $s_1(t) = 0$, the TF formulation (6) was reported in [4].

Given N_i observations $r_i^{(n)}(t)$ ($i = 0, 1$; $n = 1, 2, ..., N_i$), the Wigner-Ville spectra $\overline{W}_i(t,f)$ can be estimated by the WSs of the sample correlation operators $\hat{\mathbf{R}}_i$,

$$\widehat{W}_i(t,f) \triangleq L_{\hat{\mathbf{R}}_i}(t,f) = \frac{1}{N_i}\sum_{n=1}^{N_i} W_{r_i^{(n)}}(t,f)\,. \tag{7}$$

2.3 TF Subspace Detector

For the matched subspace detector Λ_3 in (3), the TF weight function is given by

$$\rho_3(t,f) = W_{\mathcal{S}}(t,f)\,,$$

where $W_{\mathcal{S}}(t,f) = L_{\mathbf{P}_{\mathcal{S}}}(t,f)$ is the Wigner distribution of the signal space $\mathcal{S}$ as defined in [20, 21]. In the case of a *non-sophisticated* signal space $\mathcal{S}$ [20], the space $\mathcal{S}$ can be associated with a TF region $\mathcal{R}$ such that

$$W_{\mathcal{S}}(t,f) \approx I_{\mathcal{R}}(t,f) \triangleq \begin{cases} 1 & \text{for } (t,f) \in \mathcal{R}\,, \\ 0 & \text{for } (t,f) \notin \mathcal{R}\,. \end{cases}$$

The area of $\mathcal{R}$ approximately equals the dimension of the space $\mathcal{S}$ [20, 21]. Replacing $\rho_3(t,f) = W_{\mathcal{S}}(t,f)$ by $\tilde{\rho}_3(t,f) = I_{\mathcal{R}}(t,f)$ yields the TF test statistic

$$\tilde{\Lambda}_3(r) \triangleq \langle I_{\mathcal{R}}, W_r\rangle = \iint_{\mathcal{R}} W_r(t,f)\, dt\, df\,,$$

which can also be written as the quadratic form $\tilde{\Lambda}_3(r) = \langle \widetilde{\mathbf{H}}_3 r, r\rangle$ with $\widetilde{\mathbf{H}}_3$ defined by $L_{\widetilde{\mathbf{H}}_3}(t,f) = I_{\mathcal{R}}(t,f)$. An efficient TF implementation will be discussed in Section 3.

The TF region $\mathcal{R}$ can be estimated as $\hat{\mathcal{R}}(\varepsilon) = \{(t,f) : \widehat{W}(t,f) \geq \varepsilon\}$, with $\widehat{W}(t,f) = \frac{1}{N_0+N_1}\big[N_0\widehat{W}_0(t,f) + N_1\widehat{W}_1(t,f)\big]$ (cf. (7)) and ε chosen such that $\iint_{\hat{\mathcal{R}}(\varepsilon)} dt\, df = p$ (the dimension of $\mathcal{S}$). This procedure is the TF counterpart of ML subspace identification (cf. Subsection 1.2).

These approximate TF versions of matched subspace detection and ML subspace identification are more intuitive, stable, and efficient than the original methods that require a numerically expensive and sensitive eigendecomposition.

2.4 Extended TF Subspace Detector

The TF detector $\tilde{\Lambda}_3$ can easily be generalized to the extended matched subspace detector Λ_4 in (4). We associate to each subspace $\mathcal{S}_k$ a TF region $\mathcal{R}_k$ such that $I_{\mathcal{R}_k}(t,f) \approx W_{\mathcal{S}_k}(t,f)$ and define the TF test statistic

$$\tilde{\Lambda}_4(r) = \sum_{k=1}^{K} \big[\theta_0^{(k)} - \theta_1^{(k)}\big]\, \tilde{\xi}_k \quad \text{with } \tilde{\xi}_k = \iint_{\mathcal{R}_k} W_r(t,f)\, dt\, df. \tag{8}$$

Alternatively, $\tilde{\Lambda}_4(r) = \langle \widetilde{\mathbf{H}}_4 r, r\rangle$ with $\widetilde{\mathbf{H}}_4$ defined by $L_{\widetilde{\mathbf{H}}_4}(t,f) = \sum_{k=1}^{K} \big[\theta_0^{(k)} - \theta_1^{(k)}\big]\, I_{\mathcal{R}_k}(t,f)$.

3 MULTI-WINDOW STFT IMPLEMENTATION

For jointly underspread processes $s_0(t)$, $s_1(t)$, it can be shown that the various operators occurring in the conventional and TF designed test statistics are effectively underspread. It is known [11] that any underspread operator $\mathbf{H}$ can be decomposed as $\mathbf{H} = \sum_{l=1}^{\infty} \lambda_l\, \mathbf{Q}_l$. Here, $\mathbf{Q}_l$ is an operator that performs a short-time Fourier transform (STFT) analysis with analysis window $u_l(t)$, a multiplication of the resulting STFT by the TF weight function

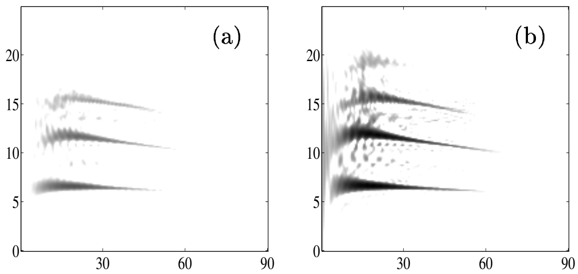

Figure 1. *Estimated Wigner-Ville spectra $\widehat{W}_i(t,f)$ calculated from (a) non-knocking (i=0) and (b) knocking (i=1) training data. Horizontal axis: crank angle (in degrees) which is proportional to time, vertical axis: frequency (in kHz).*

$L_{\mathbf{H}}(t,f)$, and finally an STFT synthesis with synthesis window $u_l(t)$ ("STFT filter") [11, 13, 22]. The calculation of the coefficients λ_l and the (orthonormal) STFT windows $u_l(t)$ is dicussed in [11]. Any underspread operator $\mathbf{H}$ can thus be represented as a linear combination of STFT filters with orthonormal windows $u_l(t)$ and identical TF weight function $L_{\mathbf{H}}(t,f)$ ("multiwindow STFT filter"). Typically, the coefficients λ_l decay fast so that it suffices to use a multiwindow STFT filter with finite (small) order L. This yields the following efficient *multiwindow STFT implementation* of the quadratic test statistic induced by $\mathbf{H}$:

$$\Lambda(r) = \langle \mathbf{H}r, r\rangle \approx \Lambda^{(L)}(r) \triangleq \left\langle \sum_{l=1}^{L} \lambda_l \mathbf{Q}_l r,\, r \right\rangle.$$

It can be shown that this can equivalently be formulated as the inner product of the WS of $\mathbf{H}$ with a multiwindow spectrogram of $r(t)$, $\Lambda^{(L)}(r) = \left\langle L_{\mathbf{H}}, \sum_{l=1}^{L} \lambda_l S_r^{(u_l)} \right\rangle$, where $S_r^{(u_l)}(t,f)$ denotes the spectrogram (i.e., squared STFT magnitude) of $r(t)$ with analysis window $u_l(t)$.

4 APPLICATION TO KNOCK DETECTION

Finally, we consider the application of the various detectors to the detection of knocking combustions in car engines. Knock detection is important since a car engine is most efficient near knocking conditions but frequent knock can damage the engine. Previous work [23, 24] showed that knock is a transient phenomenon consisting of several resonances with decreasing resonance frequencies (see Fig. 1). Thus, detectors should match this time-varying behavior.

The available observation data[3] $r_i^{(n)}(t)$ comprised 1091 pressure signals obtained from a pressure sensor mounted inside the cylinder. The signals were recorded at fixed rotation speed of 4000 rpm. The data had been manually classified into 900 non-knocking and 191 knocking combustions. This data was further split into a training set (100 non-knocking and 91 knocking signals) from which the statistical *a priori* information was estimated, and the remaining 800 non-knocking and 100 knocking signals that were used to assess the detectors' performance via empirical receiver operator characteristics (ROCs) [1, 2]. The estimated Wigner-Ville spectra are shown in Fig. 1. Fig. 2 compares the TF weight functions of some of the detectors considered. The results (ROCs) obtained with the various detectors are compared in Fig. 3 and discussed below.

4.1 Detectors Based on Complete Statistical A Priori Knowledge

Likelihood ratio detector Λ_1. The sample correlation operators $\hat{\mathbf{R}}_i$ calculated from our data were poorly conditioned. Although we used pseudo-inverses to compute $\mathbf{H}_1$ (cf. (1)), the likelihood ratio test statistic Λ_1 performed poorly (see Fig. 3(a)). This might also be (partly) due to possible non-Gaussianity of the data.

Locally optimum detector Λ_2. The design of Λ_2 does not require operator inversions (cf. (2)) and hence does not suffer from numerical instabilities. This resulted in improved performance of Λ_2 as compared to Λ_1.

TF detector $\tilde{\Lambda}_1$. Since we observed our car engine signals to be effectively underspread, we furthermore considered various TF detectors. The design of $\tilde{\Lambda}_1$ according to (5), using the estimated Wigner-Ville spectra $\widehat{W}_i(t,f)$ in (7) (see Fig. 1), merely involves divisions of functions that are easily stabilized. Compared to Λ_1, this results in a more stable detector design (cf. Fig. 2(b),(c)) and better performance. The TF detector $\tilde{\Lambda}_1$ is also seen to perform better than Λ_2.

4.2 Detectors Based on Partial Statistical A Priori Knowledge (Subspace Based Detectors)

Bandpass filter detector Λ_0. For the sake of comparison, we consider the practical, particularly simple test statistic $\Lambda_0(r) \triangleq \sum_{j=1}^{3} \int_{B_j} |R(f)|^2 df$ (with $R(f)$ the Fourier transform of $r(t)$) which is the output energy of a time-invariant band-pass filter with three pass-bands B_j matched to the dominant resonance bands of the signals. (Note, however, that systems currently in use employ only one resonance band.) This can be interpreted as a matched subspace detector with $\mathcal{S}$ the subspace of signals bandlimited to $B_1 \cup B_2 \cup B_3$. From Fig. 3(b), it is seen that Λ_0 performs worst among all subspace-based detectors considered.

Matched subspace detector Λ_3. The signal space $\mathcal{S}$ underlying Λ_3 was estimated by means of ML subspace identification (cf. Subsection 1.2) with $p = 6$ (this corresponds to allowing dimension 2 for each dominant resonance). The resulting detector Λ_3 performs better than the simple detector Λ_0 since the estimated subspace better matches transient and time-varying signals properties.

TF subspace detector $\tilde{\Lambda}_3$. The TF region $\mathcal{R}$ underlying $\tilde{\Lambda}_3$ was estimated as explained in Subsection 2.3 (with $p = 6$). Since TF thresholding is much less affected by numerical errors than the eigendecomposition required for ML subspace identification, $\tilde{\Lambda}_3$ performs partly better than Λ_3.

Extended matched subspace detector Λ_4. In our case, a meaningful determination of the subspaces $\mathcal{S}_k$ can only be done via TF space synthesis from the resonance TF regions [20, 21]. Since this implies a partial TF design, we did not apply Λ_4 but went straight for the TF detector $\tilde{\Lambda}_4$.

Extended TF subspace detector $\tilde{\Lambda}_4$. We split the estimated TF region $\mathcal{R}$ (see above) into three TF regions $\mathcal{R}_k$ corresponding to the three dominant resonances (cf. Fig. 2(f)) and computed the statistics $\tilde{\xi}_k$ in (8). Using a Kolmogorov-Smirnov test [25] with significance level 5%, we verified that the $\tilde{\xi}_k$ are reasonably well approximated by an exponential distribution with parameters $\theta_i^{(k)} = 1/\lambda_i^{(k)}$ estimated from the dominant eigenvalue of the projected sample correlation operators (see Subsection 1.3). These estimates were used to design $\tilde{\Lambda}_4$. The resulting detector performs partly better than Λ_3 but poorer than $\tilde{\Lambda}_3$.

5 CONCLUSION

We have introduced time-frequency (TF) formulations, efficient and stable TF design procedures, and an efficient TF implementation for several quadratic detectors requiring different levels of *a priori* knowledge. Conventional and TF detectors have been applied to knock detection and their performance has been compared. The proposed TF detectors were observed to be advantageous due to their robustness and numerical stability.

[3] In our experiments, all signals were discrete-time and finite-length, and hence all operators reduced to finite-size matrices. However, for the sake of consistency we shall continue using our previous notation.

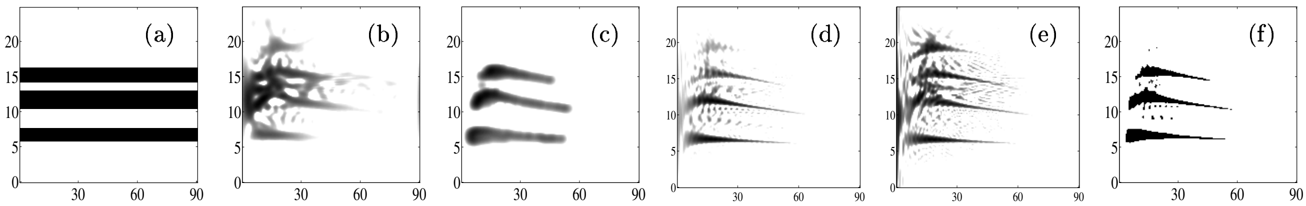

Figure 2. *TF weight functions $\rho(t,f)$ or $\tilde{\rho}(t,f)$ of (a) bandpass filter detector Λ_0, (b) likelihood ratio detector Λ_1, (c) TF detector $\tilde{\Lambda}_1$, (d) locally optimum detector Λ_2, (e) matched subspace detector Λ_3, (f) TF subspace detector $\tilde{\Lambda}_3$. (The axes are as in Fig. 1.)*

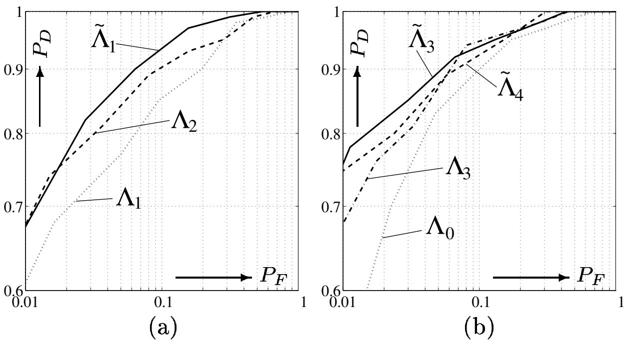

Figure 3. *ROCs of (a) detectors based on complete statistical a priori knowledge and (b) detectors based on incomplete statistical a priori knowledge (i.e., based on subspace models).*

In practice, knock detection uses vibration data obtained from an acceleration sensor mounted on the engine block (instead of the pressure data considered here). We applied the detectors discussed above to vibration data as well and obtained qualitatively similar results.

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