DOUBLY UNDERSPREAD NON-WSSUS CHANNELS: ANALYSIS AND ESTIMATION OF CHANNEL STATISTICS

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ABSTRACT

The *local scattering function* (LSF) and the generalized *channel correlation function* have recently been introduced as intuitive and useful statistical characterization of fading channels that do not satisfy the usual WSSUS assumption [1]. In this paper, we present a generalized LSF and we demonstrate that for the subclass of practically relevant *doubly underspread* non-WSSUS channels, several useful approximations and interpretations for the LSF can be given. We further propose a computationally efficient, nonstationary multiwindow LSF estimator that yields reliable estimates in the case of doubly underspread channels even for a single channel realization.

1. INTRODUCTION

Usually, the statistical characterization of fading dispersive channels has been limited to channels with *wide-sense stationary uncorrelated scattering* (WSSUS) [2, 3]. Here, scattering function and time-frequency (TF) correlation function proved useful in a variety of practical applications and theoretical developments. Since the WSSUS assumption in practice is not (exactly) satisfied (cf. the quasi-WSSUS channels in [2] and [4–7]), a practically useful and physically intuitive statistical characterization of non-WSSUS channels in terms of a *local scattering function* (LSF) and a generalized *channel correlation function* (CCF) has recently been proposed [1]. While the LSF can be interpreted as mean TF dependent scatterer power, the CCF characterizes the (nonstationary) scatterer correlation. LSF and CCF are expected to be beneficial for realistic channel simulation, performance analysis of wireless communication schemes, improved transmitter/receiver design, and for the assessment of algorithms exploiting long-term channel properties.

In this paper, we focus on *doubly underspread* (DU) non-WSSUS channels [1] and present the following new results:

- We define generalized LSFs based on the concept of "atomic" channels. We show that in the case of DU channels, all LSF definitions, while theoretically distinct, are practically equivalent.
- We demonstrate that in the case of DU channels, statistical scatterer properties change only slowly with time and frequency. During a stationarity time T_s and within a stationarity bandwidth F_s, DU channels can thus be approximated by WSSUS channels.
- We provide a Karhunen-Loève (KL) [8] type decomposition of DU channels into a set of well-structured (deterministic) atomic channels weighted by uncorrelated random coefficients.
- We introduce computationally efficient LSF estimators. A bias-variance analysis reveals that for DU channels, reliable LSF estimates can be obtained from a single channel realization.

2. LOCAL SCATTERING FUNCTION, CHANNEL CORRELATION FUNCTION, AND GENERALIZATIONS

We consider a linear, time-varying (LTV) zero-mean random channel $\mathbf{H}$ that maps the transmit signal $x(t)$ to the received signal $y(t)$ according to[1] $y(t) = (\mathbf{H}x)(t) = \int h(t,\tau)\, x(t-\tau)\, d\tau$, with $h(t,\tau)$ the impulse response of $\mathbf{H}$. Other channel descriptions are the *time-varying transfer function* $L_{\mathbf{H}}(t,f) \triangleq \int h(t,\tau)\, e^{-j2\pi f\tau} d\tau$ [2] and the *(delay-Doppler) spreading function* $S_{\mathbf{H}}(\tau,\nu) \triangleq \int h(t,\tau)\, e^{-j2\pi\nu t} dt$ [2]. Transfer function and spreading function are related by a 2-D Fourier transform.

Local Scattering Function. For a non-WSSUS channel, $L_{\mathbf{H}}(t,f)$ is a nonstationary random process with correlation function

$$R_L(t,f;\Delta t,\Delta f) \triangleq \mathbb{E}\{L_{\mathbf{H}}(t,f+\Delta f)\, L^*_{\mathbf{H}}(t-\Delta t,f)\}.$$

The LSF of $\mathbf{H}$ can be defined as [1]

$$\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) \triangleq \iint R_L(t,f;\Delta t,\Delta f)\, e^{-j2\pi(\nu\Delta t-\tau\Delta f)}\, d\Delta t\, d\Delta f\,.$$

In signal theoretical terms, the LSF can be interpreted as nonstationary power spectrum of $L_{\mathbf{H}}(t,f)$. A physical interpretation is as follows. Transmitting the signal $g_{t_0-\tau_0,f_0}(t) = g(t-t_0+\tau_0)\, e^{j2\pi f_0 t}$ (localized about the TF point $(t_0-\tau_0,f_0)$) over $\mathbf{H}$ and taking the inner product of the received signal $(\mathbf{H}g_{t_0-\tau_0,f_0})(t)$ with $g_{t_0,f_0+\nu_0}(t) = g(t-t_0)\, e^{j2\pi(f_0+\nu_0)t}$ (which is localized about $(t_0,f_0+\nu_0)$) yields a measure of the energy transfer from $(t_0-\tau_0,f_0)$ to $(t_0,f_0+\nu_0)$ effected by $\mathbf{H}$. On average, this TF energy transfer equals

$$\mathbb{E}\big\{\big|\big\langle \mathbf{H}g_{t_0-\tau_0,f_0}, g_{t_0,f_0+\nu_0}\big\rangle\big|^2\big\} = \big(\mathscr{C}_{\mathbf{H}} *_4 K_g\big)(t_0,f_0;\tau_0,\nu_0). \qquad (1)$$

Here, $*_d$ denotes d-dimensional convolution and $K_g(t,f;\tau,\nu)$ is a well concentrated smoothing kernel that depends on $g(t)$. According to (1), the locally averaged LSF the mean power[2] of scatterers causing a delay-Doppler shift (τ_0,ν_0) of transmit signal components localized about the TF point (t_0,f_0).

In the special case of $\mathbf{H}$ being WSSUS, there is $R_L(t,f;\Delta t,\Delta f) = R_{\mathbf{H}}(\Delta t,\Delta f)$ and $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) = C_{\mathbf{H}}(\tau,\nu)$, where $R_{\mathbf{H}}(\Delta t,\Delta f)$ and $C_{\mathbf{H}}(\tau,\nu)$ denote the *TF correlation function* and the *scattering function* of $\mathbf{H}$, respectively [2, 3].

Channel Correlation Function. The LSF has been complemented with the CCF, defined as [1]

$$\mathscr{A}_{\mathbf{H}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu) \triangleq \iiiint \mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)\, e^{-j2\pi(\Delta\nu t-\Delta\tau f)} \times e^{j2\pi(\Delta t\nu-\Delta f\tau)}\, dt\, df\, d\tau\, d\nu\,. \qquad (2)$$

The CCF is symmetric, assumes its maximum at the origin, and can be viewed as measure of the correlation of scatterers separated by Δt, Δf, $\Delta\tau$, and $\Delta\nu$ in time, frequency, delay, and Doppler frequency, respectively. That is, the CCF extension in the Δt and Δf direction characterizes the temporal and spectral correlation/coherence of $L_{\mathbf{H}}(t,f)$ and the CCF extension in the $\Delta\tau$ and $\Delta\nu$ direction characterizes the delay and Doppler correlation of $S_{\mathbf{H}}(\tau,\nu)$. For WSSUS channels, scatterers with different delay or Doppler are uncorrelated; correspondingly, $\mathscr{A}_{\mathbf{H}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = R_{\mathbf{H}}(\Delta t,\Delta f)\,\delta(\Delta\tau)\,\delta(\Delta\nu)$.

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[1] Integrals and sums are from $-\infty$ to ∞ unless stated otherwise.

[2] We note that in general $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ itself is not guaranteed to be real-valued and positive. This problem will be settled in Section 3.

Generalized LSF. The LSF is not guaranteed to be positive and requires the whole correlation function $R_L(t,f;\Delta t,\Delta f)$ for its computation. In the following, we define generalized LSFs (GLSF) that allow to avoid these disadvantages while maintaining an intuitive physical interpretation. These GLSFs will be seen to be important in the context of LSF estimation in Section 4.

Consider the *deterministic* "atomic" LTV channels

$$\mathbf{G}_{t,f}^{\tau,\nu} = \mathbf{S}_{\tau,\nu}\mathbf{S}_{t,f}\mathbf{G}\mathbf{S}_{t,f}^{+}, \tag{3}$$

where $\mathbf{S}_{t,f}$ denotes the TF shift operator (defined by $(\mathbf{S}_{t_0,f_0}x)(t) = x(t-t_0)\,e^{j2\pi f_0 t}$) and $\mathbf{G}$ is a normalized ($\|\mathbf{G}\|=1$) LTV prototype system whose transfer function $L_{\mathbf{G}}(t,f)$ (and thus TF pass region) is smooth and localized about the origin of the TF plane. Correspondingly, the TF pass region of $\mathbf{S}_{t,f}\mathbf{G}\mathbf{S}_{t,f}^{+}$ is localized about the TF point (t,f). Hence, the atomic channel $\mathbf{G}_{t,f}^{\tau,\nu}$ extracts transmit signal components localized about (t,f) and shifts them by τ in time and by ν in frequency. The atomic channels (3) are basic building blocks for an arbitrary LTV channel $\mathbf{H}$ in the sense that

$$\mathbf{H} = \iiiint \mathscr{H}^{(\mathbf{G})}(t,f;\tau,\nu)\,\mathbf{G}_{t,f}^{\tau,\nu}\,dt\,df\,d\tau\,d\nu\,. \tag{4}$$

The coefficients in this weighted superposition are given by

$$\mathscr{H}^{(\mathbf{G})}(t,f;\tau,\nu) = \big\langle \mathbf{H},\mathbf{G}_{t,f}^{\tau,\nu}\big\rangle = \big\langle L_{\mathbf{H}}, L_{\mathbf{G}_{t,f}^{\tau,\nu}}\big\rangle$$
$$= \iint L_{\mathbf{H}}(t',f')\,L_{\mathbf{G}}^{*}(t'-t,f'-t)\,e^{j2\pi(t'\nu-f'\tau)}\,dt'\,df'\,. \tag{5}$$

From the above discussion, it follows that the GLSF

$$\mathscr{S}_{\mathbf{H}}^{(\mathbf{G})}(t,f;\tau,\nu) \triangleq \mathbb{E}\big\{\big|\mathscr{H}^{(\mathbf{G})}(t,f;\tau,\nu)\big|^2\big\} \tag{6}$$

can be interpreted as mean power of scatterers causing a delay-Doppler shift (τ,ν) of signal components localized about the TF point (t,f). In fact, (6) can be viewed as 4-D expected spectrogram [8] of the channel that is related to the LSF as

$$\mathscr{S}_{\mathbf{H}}^{(\mathbf{G})}(t,f;\tau,\nu) = (\mathscr{C}_{\mathbf{H}} *_4 \mathscr{W}_{\mathbf{G}})(t,f;\tau,\nu)\,,$$

where $\mathscr{W}_{\mathbf{G}}(t,f;\tau,\nu) \triangleq L_{\mathbf{G}}(t,f)\,S_{\mathbf{G}}^{*}(\tau,\nu)\,e^{j2\pi(f\tau-t\nu)}$ is the transfer Rihczazek distribution of $\mathbf{G}$ [9]. With the assumptions stated above, both $L_{\mathbf{G}}(t,f)$ and $S_{\mathbf{G}}(\tau,\nu)$ are reasonably concentrated about the origin so that $\mathscr{S}_{\mathbf{H}}^{(\mathbf{G})}(t,f;\tau,\nu)$ is a slightly smoothed version of the LSF. Since $L_{\mathbf{G}}(t,f)$ and $S_{\mathbf{G}}(\tau,\nu)$ are a 2-D Fourier transform pair, the amount of smoothing in the TF coordinates (t,f) and the delay-Doppler coordinates (τ,ν) can be traded via choice of $\mathbf{G}$. The GLSF in (6) is always real-valued and positive and involves only "local" channel properties (cf. (5)).

A simple extension of (6) is obtained by using K different (normalized) prototype systems $\mathbf{G}_k$, $k=1,\ldots,K$, leading to the GLSF

$$\mathscr{C}_{\mathbf{H}}^{(\Phi)}(t,f;\tau,\nu) \triangleq \sum_{k=1}^{K}\gamma_k\,\mathscr{S}_{\mathbf{H}}^{(\mathbf{G}_k)}(t,f;\tau,\nu) \tag{7}$$
$$= (\mathscr{C}_{\mathbf{H}} *_4 \Phi)(t,f;\tau,\nu)\,, \tag{8}$$

where $\Phi(t,f;\tau,\nu) = \sum_{k=1}^{K}\gamma_k\mathscr{W}_{\mathbf{G}_k}(t,f;\tau,\nu)$. The GLSF in (7) can be interpreted as expected multiwindow spectrogram of $\mathbf{H}$. It will be nonnegative iff $\gamma_k>0$, $k=1,\ldots,K$. Using (8), the Fourier transform $\mathscr{A}_{\mathbf{H}}^{(\Phi)}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ of $\mathscr{C}_{\mathbf{H}}^{(\Phi)}(t,f;\tau,\nu)$ can be shown to equal

$$\mathscr{A}_{\mathbf{H}}^{(\Phi)}(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = \mathscr{A}_{\mathbf{H}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\,\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\,, \tag{9}$$

where $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ is the Fourier transform of $\Phi(t,f;\tau,\nu)$. Eq. (9) reflects the fact that the smoothing of $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ in (8) will be stronger if $\Phi(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ is a lowpass function whose Fourier transform $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ is concentrated about the origin.

3. DOUBLY UNDERSPREAD CHANNELS

In [1], the notion of *doubly underspread* (DU) channels was introduced to characterize non-WSSUS channels that are both *dispersion underspread* and *correlation underspread*. A channel $\mathbf{H}$ is dispersion underspread if $\iint \mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)\,dt\,df = \mathbb{E}\{|S_{\mathbf{H}}(\tau,\nu)|^2\}$ is concentrated about the origin, meaning that $\mathbf{H}$ causes only small time-delays and/or small Doppler frequency shifts. Note that by the Fourier duality of $\tau\leftrightarrow\Delta f$ and $\nu\leftrightarrow\Delta t$ (cf. (2)), a small extension of the LSF in the τ and ν direction implies a large extension of the CCF in the Δf and Δt, direction respectively. That is, a small maximum delay $\tau_{\max}$ (maximum Doppler $\nu_{\max}$) results in a large coherence bandwidth $F_c = 1/\tau_{\max}$ (coherence time $T_c \triangleq 1/\nu_{\max}$).

A channel $\mathbf{H}$ is correlation underspread if its CCF is concentrated about the origin, meaning that only scatterers that are "close" in time, frequency, delay, and Doppler are correlated. To measure the spread of the CCF, we use the weighted integrals

$$s_{\mathbf{H}}^{(w)} \triangleq \iiiint w(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\,\big|\mathscr{A}_{\mathbf{H}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\big|\,d\Delta t\,d\Delta f\,d\Delta\tau\,d\Delta\nu\,.$$

Here, $w(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\ge 0$ is a nonnegative weight function satisfying $w(0,0;0,0)=1$ and penalizing CCF contributions off the origin. Thus, $s_{\mathbf{H}}^{(w)}$ is small for DU channels. We next discuss important consequences of the DU property for the (G)LSF.

Equivalence. While GLSFs with different $\Phi(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ are formally different, in the DU case all GLSF are approximately equivalent. In particular, using (9) one can prove the bound

$$\big|\mathscr{C}_{\mathbf{H}}^{(\Phi_1)}(t,f;\tau,\nu) - \mathscr{C}_{\mathbf{H}}^{(\Phi_2)}(t,f;\tau,\nu)\big| \le s_{\mathbf{H}}^{(w)}$$

with $w(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = \hat{\Phi}_1(\Delta t,\Delta f;\Delta\tau,\Delta\nu) - \hat{\Phi}_2(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$. If $\mathbf{H}$ is correlation underspread and $\hat{\Phi}_1(\Delta t,\Delta f;\Delta\tau,\Delta\nu) \approx \hat{\Phi}_2(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ on the effective support of $\big|\mathscr{A}_{\mathbf{H}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\big|$, then the expression on the right-hand side will be approximately zero and thus

$$\mathscr{C}_{\mathbf{H}}^{(\Phi_1)}(t,f;\tau,\nu) \approx \mathscr{C}_{\mathbf{H}}^{(\Phi_2)}(t,f;\tau,\nu)\,.$$

In the important special case $\mathscr{C}_{\mathbf{H}}^{(\Phi_2)}(t,f;\tau,\nu) = \mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ (i.e., $\hat{\Phi}_2(\Delta t,\Delta f;\Delta\tau,\Delta\nu)=1$), the approximation

$$\mathscr{C}_{\mathbf{H}}^{(\Phi)}(t,f;\tau,\nu) \approx \mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) \tag{10}$$

holds if $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\approx 1$ on the effective support of the CCF.

Realvaluedness and Positivity. While the LSF $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ is not guaranteed to be real-valued and positive, the GLSF $\mathscr{S}_{\mathbf{H}}^{(\mathbf{G})}(t,f;\tau,\nu)$ is positive but involves a somewhat arbitrary prototype $\mathbf{G}$. The equivalence $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) \approx \mathscr{S}_{\mathbf{H}}^{(\mathbf{G})}(t,f;\tau,\nu)$ for DU channels suggests that here the imaginary and negative parts of $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ are negligible. Let $\mathscr{C}_{\mathbf{H}}^{(+)}(t,f;\tau,\nu)$ denote the positive real part of $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$. It can then be shown that

$$\big|\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) - \mathscr{C}_{\mathbf{H}}^{(+)}(t,f;\tau,\nu)\big| \le \pi\, s_{\mathbf{H}}^{(w_1)} + \min_{\mathbf{G}} s_{\mathbf{H}}^{(w_2)}$$

Here, $w_1(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = |\Delta t|\,|\Delta f|\,|\Delta\tau|\,|\Delta\nu|$ and $w_2(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = \big|1-\hat{\Phi}_{\mathbf{G}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\big|$ with $\hat{\Phi}_{\mathbf{G}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ denoting the Fourier transform of $\mathscr{W}_{\mathbf{G}}(t,f;\tau,\nu)$. If $\mathbf{H}$ is correlation underspread, $s_{\mathbf{H}}^{(w_1)}$ is small and $s_{\mathbf{H}}^{(w_2)}$ can be made small by suitable choice of $\mathbf{G}$. Thus,

$$\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) \approx \mathscr{C}_{\mathbf{H}}^{(+)}(t,f;\tau,\nu)\,. \tag{11}$$

We conclude that for DU channels the LSF is effectively positive.

TF Smoothness and WSSUS Approximation. According to (2), $t\leftrightarrow\Delta\nu$ and $f\leftrightarrow\Delta\tau$ are Fourier dual variables. Thus, the LSF variation with respect to t, f is determined by the CCF extension in the $\Delta\tau$, $\Delta\nu$ direction. This can further be seen, e.g., from the bounds

$$\left|\frac{\partial^k\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)}{\partial t^k}\right| \le (2\pi)^k s_{\mathbf{H}}^{(w_1)}, \qquad \left|\frac{\partial^k\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)}{\partial f^k}\right| \le (2\pi)^k s_{\mathbf{H}}^{(w_2)},$$

where $w_1(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = |\Delta\nu|^k$ and $w_2(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = |\Delta\tau|^k$. An alternative bound on the rate of variation of the LSF is

$$\big|\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)-\mathscr{C}_{\mathbf{H}}(t_0,f_0;\tau,\nu)\big| \le 2\pi\big[|t-t_0|\,s_{\mathbf{H}}^{(w_1)}+|f-f_0|\,s_{\mathbf{H}}^{(w_2)}\big]. \quad (12)$$

Both types of inequalities show that for DU channels (where $s_{\mathbf{H}}^{(w_1)}$ and $s_{\mathbf{H}}^{(w_2)}$ are small), the LSF is a smooth function of t and f. Let us define a *stationarity time* $T_{\mathrm{s}} \triangleq 1/s_{\mathbf{H}}^{(w_1)}$ and a *stationarity bandwidth* $F_{\mathrm{s}} \triangleq 1/s_{\mathbf{H}}^{(w_2)}$. According to (12) and (11), there is $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) \approx \mathscr{C}_{\mathbf{H}}(t_0,f_0;\tau,\nu) \approx \mathscr{C}_{\mathbf{H}}^{(+)}(t_0,f_0;\tau,\nu)$ for $|t-t_0| \ll T_{\mathrm{s}}$ and $|f-f_0| \ll F_{\mathrm{s}}$. Thus, within the time interval $[t_0,t_0+T_{\mathrm{s}}]$ and the frequency band $[f_0,f_0+F_{\mathrm{s}}]$, the LSF is (effectively) constant and the DU channel can be approximated by a WSSUS channel with scattering function $\mathscr{C}_{\mathbf{H}}^{(+)}(t_0,f_0;\tau,\nu)$. We note that for correlation underspread channels, $s_{\mathbf{H}}^{(w_1)}$ and $s_{\mathbf{H}}^{(w_2)}$ are small and thus T_s and F_s are rather large (for a WSSUS channel, $T_{\mathrm{s}} = F_{\mathrm{s}} = \infty$).

Example. We consider a simple example dealing with an indoor WLAN type scenario. We assume a carrier frequency of $f_{\mathrm{c}} = 5.4\,\mathrm{GHz}$, a maximum receiver velocity of $v_0 = 1\,\mathrm{m/s}$, and a maximum path length between transmitter and receiver of $d = 30\,\mathrm{m}$. This corresponds to a maximum delay and Doppler of $\tau_{\max} = \frac{d}{c} = 100\,\mathrm{ns}$ and $\nu_{\max} = \frac{v_0}{c} f_{\mathrm{c}} = 18\,\mathrm{Hz}$, respectively. Correspondingly, $T_{\mathrm{c}} = 55.6\,\mathrm{ms}$ and $F_{\mathrm{c}} = 10\,\mathrm{MHz}$. With $\tau_{\max}\nu_{\max} = 1.8\cdot10^{-6} \ll 1$, we conclude that the channel is dispersion underspread. Scatterer correlation usually results from reflections by the same physical object. Assuming the maximum width and angular spread of the scatterers to be $w = 0.3\,\mathrm{m}$ and $\delta = 1^\circ$, respectively, results in $\Delta\tau_{\max} = \frac{w}{c} = 1\,\mathrm{ns}$ and $\Delta\nu_{\max} = \nu_{\max}\sin(\delta) \approx 0.31\,\mathrm{Hz}$. Assuming $s_{\mathbf{H}}^{(w_1)} \approx \Delta\nu_{\max}$, $s_{\mathbf{H}}^{(w_2)} \approx \Delta\tau_{\max}$ results in $T_{\mathrm{s}} = 3.2\,\mathrm{s}$ and $F_{\mathrm{s}} = 1\,\mathrm{GHz}$. With the effective CCF support of $T_c F_c \Delta\tau_{\max}\Delta\nu_{\max} = 1.72\cdot10^{-4} \ll 1$, it follows that the channel is correlation underspread and thus DU.

KL-Type Representation. Non-WSSUS channels are intricate objects in that their transfer characteristics vary with time and their statistics is nonstationary. A canonical representation that decouples the randomness and the TF transfer characteristics in the spirit of a KL expansion [8] is thus highly desirable. In fact, under certain conditions regarding the "atom" $\mathbf{G}$ and the sampling periods T_0, F_0, τ_0, ν_0, any channel $\mathbf{H}$ allows for the decomposition (cf. (3))

$$\mathbf{H} = \sum_n\sum_k\sum_m\sum_l \eta_{n,k}^{m,l}\,\mathbf{G}_{nT_0,kF_0}^{m\tau_0,l\nu_0}, \qquad \text{with} \qquad \eta_{n,k}^{m,l} = \big\langle \mathbf{H}, \mathbf{\Gamma}_{nT_0,kF_0}^{m\tau_0,l\nu_0}\big\rangle, \quad (13)$$

where $\mathbf{\Gamma}$ is a prototype LTV channel that is in a certain sense dual to $\mathbf{G}$ [10]. Note that (13) is a discrete version of (4). In the case of a DU channel, one can show that the correlations of the random coefficients $\eta_{n,k}^{m,l}$ in (13) satisfy the approximation

$$\mathbb{E}\big\{\eta_{n,k}^{m,l}\,\big(\eta_{n+n',k+k'}^{m+m',l+l'}\big)^*\big\} \approx \mathscr{C}_{\mathbf{H}}(nT_0,kF_0;m\tau_0,l\nu_0)\,\delta_{n'}\,\delta_{k'}\,\delta_{m'}\,\delta_{l'}\,,$$

i.e., the coefficients $\eta_{n,k}^{m,l}$ are approximately uncorrelated and their mean power is equal to (suitable samples of) the LSF. In fact, the errors incurred by this approximation are bounded as

$$\Big|\mathbb{E}\big\{\big|\eta_{n,k}^{m,l}\big|^2\big\} - \mathscr{C}_{\mathbf{H}}(nT_0,kF_0;m\tau_0,l\nu_0)\Big| \le s_{\mathbf{H}}^{(w_1)}.$$

and

$$\Big|\mathbb{E}\big\{\eta_{n,k}^{m,l}\,\big(\eta_{n+n',k+k'}^{m+m',l+l'}\big)^*\big\}\Big| \le s_{\mathbf{H}}^{(w_2)}, \qquad n',k',m',l' \neq 0\,.$$

Here, $w_1(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = \big|1-\hat{\Phi}_{\mathbf{\Gamma}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\big|$, $w_2(\Delta t,\Delta f;\Delta\tau,\Delta\nu) = |\hat{\Phi}_{\mathbf{\Gamma}}(\Delta t-n'T_0,\Delta f-k'F_0;\Delta\tau-m'\tau_0,\Delta\nu-l'\nu_0)|$. In the case of a DU channel, $s_{\mathbf{H}}^{(w_1)}$ and $s_{\mathbf{H}}^{(w_2)}$ can both be made small by suitable choice of $\mathbf{G}$, $\mathbf{\Gamma}$, T_0, F_0, τ_0, ν_0 [10].

4. LSF ESTIMATION

We next consider estimation of the LSF of a non-WSSUS channel. Since realizations of the channel impulse response or transfer function can never be observed directly, we assume that a noisy realization $\hat{\mathbf{H}} = \mathbf{H}+\mathbf{E}$, (i.e., $\hat{h}(t,\tau) = h(t,\tau)+e(t,\tau)$ or $\hat{L}_{\mathbf{H}}(t,f) = L_{\mathbf{H}}(t,f)+L_{\mathbf{E}}(t,f)$) of the channel has been measured. Furthermore, we make the simplifying assumption that $e(t,\tau)$ and $L_{\mathbf{E}}(t,f)$ are stationary and white with known intensity σ^2.

The LSF estimator we propose is an extension of the nonstationary spectral estimators in [8, 11]:

$$\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu) \triangleq \sum_{k=1}^{K}\gamma_k\,\big|\hat{\mathscr{H}}^{(\mathbf{G}_k)}(t,f;\tau,\nu)\big|^2 - \sigma^2. \quad (14)$$

Here, $\hat{\mathscr{H}}^{(\mathbf{G}_k)}(t,f;\tau,\nu)$ is computed according to (5), with $\mathbf{H}$ replaced by $\hat{\mathbf{H}}$. We assume $\mathrm{tr}\{\mathbf{G}_k\} = 1$ and $\sum_{k=1}^K\gamma_k = 1$. Subtraction of σ^2 in (14) can be shown to result in partial bias compensation. Note that (14) is just (7) without expectation. The LSF estimator in (14) can alternatively be rewritten as

$$\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu) = (\mathscr{W}_{\hat{\mathbf{H}}} *_4 \Phi)(t,f;\tau,\nu) - \sigma^2 \quad (15)$$

with $\Phi(t,f;\tau,\nu) = \sum_{k=1}^K\gamma_k\mathscr{W}_{\mathbf{G}_k}(t,f;\tau,\nu)$. Since it can be demonstrated that $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) = \mathbb{E}\big\{\mathscr{W}_{\mathbf{H}}(t,f;\tau,\nu)\big\}$, (15) shows that the estimate $\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)$ replaces expectation with a local averaging. We shall next provide a bias-variance analysis for $\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)$.

Bias. Starting with (15), we obtain

$$\begin{aligned}\mathbb{E}\big\{\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)\big\} &= \mathbb{E}\big\{(\mathscr{W}_{\hat{\mathbf{H}}} *_4 \Phi)(t,f;\tau,\nu)\big\} - \sigma^2\\ &= (\mathscr{C}_{\mathbf{H}} *_4 \Phi)(t,f;\tau,\nu) = \mathscr{C}_{\mathbf{H}}^{(\Phi)}(t,f;\tau,\nu)\,.\end{aligned}$$

Here, we used the fact that $\mathscr{C}_{\mathbf{E}}(t,f;\tau,\nu) = \sigma^2$ and that $\mathbf{H}$ and $\mathbf{E}$ are uncorrelated. Thus, the estimator bias equals

$$\begin{aligned}B(t,f;\tau,\nu) &\triangleq \mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) - \mathbb{E}\big\{\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)\big\}\\ &= \mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) - \mathscr{C}_{\mathbf{H}}^{(\Phi)}(t,f;\tau,\nu).\end{aligned}$$

According to (10), $B(t,f;\tau,\nu) \approx 0$ if $\mathbf{H}$ is a DU channel and if $\Phi(t,f;\tau,\nu)$ is designed such that $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu) \approx 1$ on the effective support of $\mathscr{A}_{\mathbf{H}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$. This claim is supported by the following expression for the overall bias squared:

$$\begin{aligned}\iiiint \big|B(t,f;\tau,\nu)\big|^2\,dt\,df\,d\tau\,d\nu &= \iiiint \big|\mathscr{A}_{\mathbf{H}}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\big|^2\\ &\quad\times \big|1-\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)\big|^2\,d\Delta t\,d\Delta f\,d\Delta\tau\,d\Delta\nu\,.\end{aligned}$$

Variance. The derivation of the estimator variance builds upon (14) and the assumption that channel and noise are Gaussian. Using Isserlis' theorem [12], one obtains after some manipulations

$$\begin{aligned}V^2(t,f;\tau,\nu) &\triangleq \mathbb{E}\big\{\big|\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)\big|^2\big\} - |B(t,f;\tau,\nu)|^2\\ &= \sum_{k=1}^K\sum_{k'=1}^K\gamma_k\gamma_{k'}^*\big|(\mathscr{C}_{\mathbf{H}} *_4 \mathscr{W}_{\mathbf{G}_k,\mathbf{G}_{k'}})(t,f;\tau,\nu)\big|^2\\ &\quad + 2\sigma^2\mathscr{C}_{\mathbf{H}}^{(\Phi')}(t,f;\tau,\nu) + \sigma^4\|\Phi\|^2 \quad (16)\\ &\le \|\Phi\|^2\big(\|\mathscr{C}_{\mathbf{H}}\|+\sigma^2\big)^2, \quad (17)\end{aligned}$$

with $\Phi'(t,f;\tau,\nu) = \sum_{k=1}^K|\gamma_k|^2\mathscr{W}_{\mathbf{G}_k}(t,f;\tau,\nu)$. Eq. (16) shows that for those $(t,f;\tau,\nu)$ where $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ (and thus $\mathscr{C}_{\mathbf{H}}^{(\Phi')}(t,f;\tau,\nu)$) is large, also the variance of $\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)$ will be large. Inequality

(17) shows that the estimator variance is upper bounded in terms of the squared norm of the smoothing kernel $\Phi(t,f;\tau,\nu)$. Furthermore, the $(t,f;\tau,\nu)$-dependent variance part $\tilde{V}^2(t,f;\tau,\nu) = V^2(t,f;\tau,\nu) - \sigma^4\|\Phi\|^2$ can be shown to equal on overall

$$\iiiint \tilde{V}^2(t,f;\tau,\nu)\,dt\,df\,d\tau\,d\nu = \|\Phi\|^2\left(\|\mathscr{C}_{\mathbf{H}}\|^2 + 2\sigma^2\rho^2_{\mathbf{H}}\right),$$

where $\rho^2_{\mathbf{H}} \triangleq \iiiint \mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)\,dt\,df\,d\tau\,d\nu$. Since

$$\|\Phi\|^2 = \|\hat{\Phi}\|^2 = \iiiint |\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)|^2\,d\Delta t\,d\Delta f\,d\Delta\tau\,d\Delta\nu,$$

and $\hat{\Phi}(0,0;0,0) = 1$, it follows that in order to achieve small estimator variance, the support of $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu)$ about the origin should be small. In general, this goal is in conflict with the requirement for a small bias, $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu) \approx 1$. However, for DU channels, the CCF is effectively supported within a hypercube $\mathscr{R}$ whose volume A is much less than one, $A \ll 1$; using $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu) \approx 1$ within $\mathscr{R}$ and $\hat{\Phi}(\Delta t,\Delta f;\Delta\tau,\Delta\nu) \approx 0$ else allows to achieve a small bias and, due to $\|\hat{\Phi}\|^2 \approx A$, a small variance. We conclude that DU channels allow for reliable LSF estimation.

Design Guidelines. We finally provide some guidelines for the design of the LSF estimator in (14). The individual terms in (14) require computation of (cf. (5))

$$\mathscr{H}^{(\mathbf{G}_k)}(t,f;\tau,\nu) = \iint L_{\hat{\mathbf{H}}}(t',f')\,L^*_{\mathbf{G}_k}(t'-t,f'-t)\,e^{j2\pi(t'\nu-f'\tau)}\,dt'df'.$$

For each TF analysis point (t,f), this requires i) multiplication of the measured transfer function $L_{\hat{\mathbf{H}}}(t',f')$ with a version of the "window" $L_{\mathbf{G}_k}(t',f')$ that is shifted to (t,f) and ii) a 2-D Fourier transform of this product. To make these computations feasible, we propose a simple separable window $L_{\mathbf{G}_k}(t,f) = u_k(t)\,\tilde{u}_k(f)$ whose support is contained within the rectangle $[-T/2,T/2]\times[-F/2,F/2]$. A practically useful choice for $u_k(t)$, $\tilde{u}_k(f)$ is given by the prolate spheroidal wave functions $p_i^{(T)}(t)$ of duration T (see e.g. [8]). We use I prolate spheroidal wave functions in the time direction, $p_i^{(T)}(t)$, $i = 1,\ldots,I$, and J prolate spheroidal wave functions in the frequency direction, $\sqrt{\frac{T}{F}}\,p_j^{(T)}\!\left(\frac{fT}{F}\right)$, $j = 1,\ldots,J$, to obtain a total number of IJ TF windows $L_{\mathbf{G}_k}(t,f) = \sqrt{\frac{T}{F}}\,p_i^{(T)}(t)\,p_j^{(T)}\!\left(\frac{fT}{F}\right)$, $k = (i-1)J + j$. The choice of the duration T and bandwidth F of $\mathbf{G}_k$ is directed by conflicting requirements: TF should be small such that the nonstationary channel statistics are not averaged out and bias is kept small; on the other hand, large TF implies larger regions over which the statistics of $L_{\hat{\mathbf{H}}}(t,f)$ are averaged, thereby leading to smaller estimation variance. A reasonable compromise is to choose the *effective* duration/bandwidth of the temporal/spectral windows equal to T_s and F_s, respectively (leading to $T = cT_s$ and $F = cF_s$ with $c = 2\ldots3$). A similar bias/variance tradeoff applies to the choice of I and J (more windows lead to increased bias but reduced variance). Generalizing arguments in [11], it can be shown that a judicious choice is given by $I \approx \sqrt{T_s/T_c}$ and $J \approx \sqrt{F_s/F_c}$.

5. NUMERICAL RESULTS

Simulated Channel. We first consider LSF estimation[3] for a synthetic non-WSSUS channel that was simulated according to

$$h(t,\tau) = \sum_{p=1}^{P} \alpha_p(t)\,h_p(t,\tau).$$

Here, the $h_p(t,\tau)$, $p = 1,\ldots,P$ are impulse responses of P uncorrelated WSSUS channels with distinct scattering functions $C_{\mathbf{H}_p}(\tau,\nu)$

[3]In the numerical simulations, we used discretized versions of $h(t,\tau)$, $L_{\mathbf{H}}(t,f)$, and $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ obtained via simple sampling arguments.

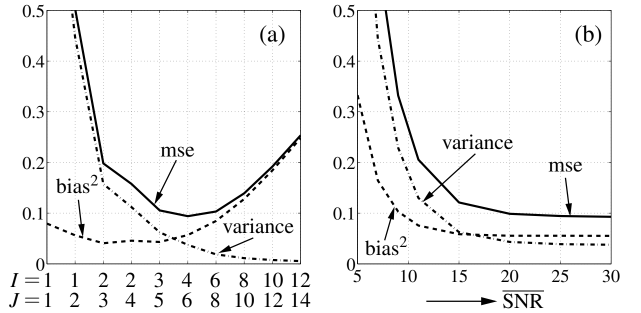

Figure 1: *LSF estimator performance (a) versus number of time windows I and number of frequency windows J and (b) versus mean SNR (in dB).*

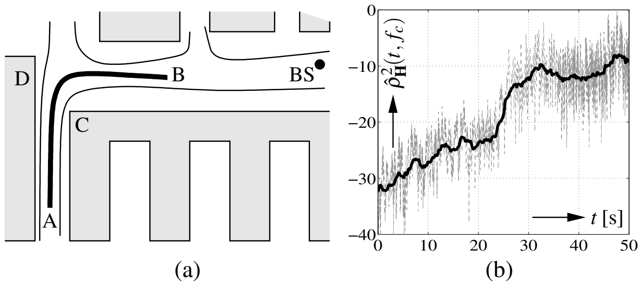

Figure 2: *(a) Measurement scenario (dark gray indicates buildings); (b) estimated TF path loss $\hat{\rho}^2_{\mathbf{H}}(t,f_c)$ (thick black, in dB) and instantaneous path loss (thin gray).*

and the $\alpha_p(t)$ are deterministic time-varying weights. The LSF of this channel is given by $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu) = \sum_{p=1}^{P}|\alpha_p(t)|^2 C_{\mathbf{H}_p}(\tau,\nu)$. To ensure a DU channel, the weights $\alpha_p(t)$ change only slowly with time. Note that $\mathscr{C}_{\mathbf{H}}(t,f;\tau,\nu)$ is independent of f, corresponding to uncorrelated channel taps.

For each simulated channel realization, we computed LSF estimates according to Section 4 using various numbers I and J of prolate spheroidal wave functions for the temporal and spectral windowing. Fig. 1(a) shows the normalized overall squared bias, variance, and mean-square-error (MSE) of these estimates for the noise-free case (computed by averaging over 40 simulation runs). It is seen that while the bias is quite small for small I, J, variance and thus MSE is very high. In contrast, variance is very small for large I, J; here, however, the MSE is degraded by a significant bias. An optimal balance of bias and variance is seen to be achieved for $I = 4$, $J = 6$. Here, the normalized MSE is seen to be about 10%, verifying that reliable LSF estimation based on single channel measurements is possible. Fig. 1(b) shows the dependence of overall squared bias, variance, and MSE on the mean SNR (i.e., ratio of average path loss and σ^2) with $I = 4$, $J = 6$ fixed. It is seen that for mean SNRs larger than 15 dB, the MSE of the LSF estimate is almost identical to that of the noise-free case.

Measured Channel. We next applied our LSF estimator to a channel measured[4] in the course of the METAMORP project [13]. at a carrier frequency of $f_c = 1792\,\text{MHz}$, impulse response snapshots $h(kT_{\text{rep}}, l\tau_0)$, $k = 1,\ldots,2160$, $l = 1,\ldots,64$, were recorded every $T_{\text{rep}} = 49.152$ ms with measurement bandwidth $B = 1/\tau_0 = 10\,\text{MHz}$. The total measurement duration was 50.33 s. The suburban propagation scenario is shown in Fig. 2(a). The base station is labeled 'BS' and the mobile moved from position 'A' around a corner (la-

[4]Courtesy of T-NOVA Deutsche Telekom Innovationsgesellschaft mbH (Technologiezentrum Darmstadt, Germany). We are grateful to I. Gaspard and M. Steinbauer for providing us with the measurement data.

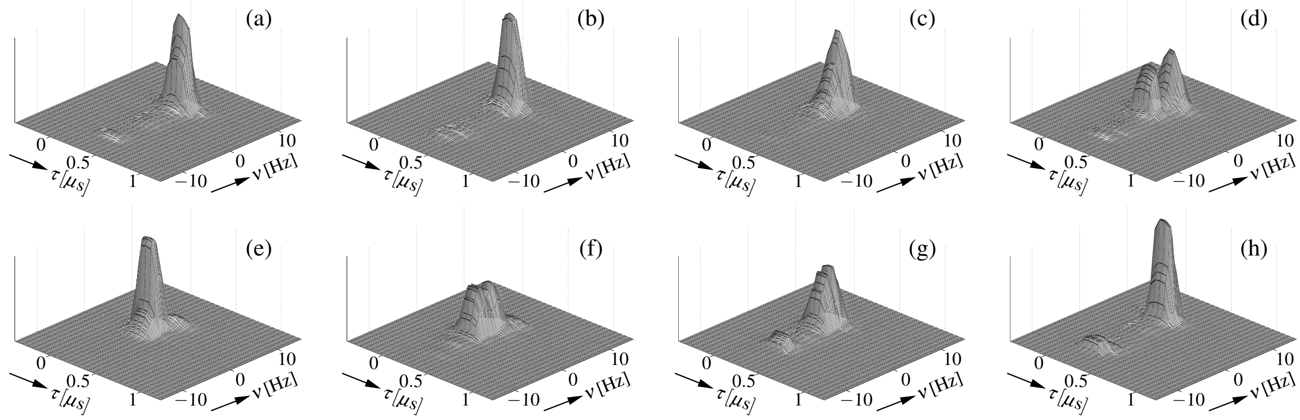

Figure 3: *Eight snapshots of the (normalized) LSF estimate* $\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)$ *of the measured channel with* $f = f_c$ *fixed and (a)* $t = 3\,s$*, (b)* $t = 9\,s$*, (c)* $t = 18\,s$*, (d)* $t = 22\,s$*, (e)* $t = 25\,s$*, (f)* $t = 28\,s$*, (g)* $t = 32\,s$*, (h)* $t = 45\,s$*.*

beled 'C') to position 'B' with a constant speed of 1.6 m/s (corresponding to $\nu_{\max} = 9.56\,\text{Hz}$).

We first subtracted a quite weak channel mean from the measured impulse response. Then, using a rough LSF estimate and physical considerations, we estimated $T_c \approx 100\,\text{ms}$, $F_c \approx 1.5\,\text{MHz}$, $T_s \approx 3\,\text{s}$, and $F_s \approx 24\,\text{MHz}$ (thus, this channel is DU). An improved LSF estimate $\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)$ was then obtained according to Section 4. We used $I = 6 \approx \sqrt{T_s/T_c}$ temporal windows of length $T = 6.3\,\text{s}$ (effective duration $\approx T_s$), and $J = 4 \approx \sqrt{F_s/F_c}$ spectral windows of maximum possible bandwidth $F = B = 10\,\text{MHz}$. Fig. 3 shows $\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)$ for fixed $f = f_c$ (as a consequence of $F_s > B$, $\widehat{\mathscr{C}}_{\mathbf{H}}(t,f;\tau,\nu)$ was virtually independent of f within the measurement band) and several time instants. For better visibility, all LSF estimates where normalized by the reciprocal of the time-varying path loss $\hat{\rho}^2_{\mathbf{H}}(t,f_c) = \iint \widehat{\mathscr{C}}_{\mathbf{H}}(t,f_c;\tau,\nu)\,d\tau\,d\nu$ (shown in Fig. 2(b) together with the "instantaneous path loss" $\int_\tau |h(t,\tau|^2\,d\tau)$.

The LSF estimate can be directly related to the physical propagation scenario in Fig. 2(a) and to the various phases of the mobile's movement. Initially ($t = 0-20\,\text{s}$, Fig. 3(a)-(c)), the mobile moves through a street towards the corner. Here, shadowing causes a large path loss (cf. Fig. 2(b)); furthermore, the Doppler frequencies of the dominant scatterers close to $\nu_{\max}$, which correspond to angle-of-arrivals (AOA) $\approx 0°$, indicate that the majority of paths arrives through the street ahead of the mobile. Since the mobile gets closer to the base station, the delay of the dominant scatterers drifts by about $0.1\,\mu\text{s}$ from $\approx 0.4\,\mu\text{s}$ to $\approx 0.3\,\mu\text{s}$ (corresponding to the covered distance of 40 m). Then, a transition phase starts that lasts from $t = 20$ to $t = 30$ (Fig. 3(d)-(f)). At the corner, the AOA and Doppler spread widen up and eventually a line-of-sight component with Doppler frequencies close to 0 Hz (AOA $\approx 70-80°$) appears (Fig.3(e),(f); cf. also Fig. 2(b)). As the mobile turns right after the corner and approaches the base station ($t = 30-50\,\text{s}$, Fig. 3(g),(h)), the AOA of the dominant line-of-sight component is $0°$, resulting in a Doppler frequency $\approx \nu_{\max}$. The delays of this dominant path drift by $0.1\,\mu\text{s}$ from $\approx 0.25\,\mu\text{s}$ to $\approx 0.15\,\mu\text{s}$. A second, weaker component with Doppler frequency $-\nu_{\max}$ (AOA 180°) can be attributed to reflections by the building labeled 'D' in Fig. 2(a).

6. CONCLUSIONS

In this paper, we introduced a generalized local scattering function (LSF) and applied the LSF and the channel correlation function (CCF) to the practically important class of doubly underspread (DU) non-WSSUS channels. Such channels are characterized by i) small delay-Doppler shifts and ii) a CCF that is concentrated about the origin (meaning that only closely spaced scatterers are correlated). The LSF of DU channels features several (approximate) properties that simplify its interpretation and practical application. Finally, we established a multi-window LSF estimator that is computationally efficient, involves only local channel properties, and allows a simple bias-variance trade-off. For DU non-WSSUS channels, this estimator yields reliable results even if only a single channel measurement is available.

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