Multi-user Channel Estimation in OFDMA Uplink Systems Based on Irregular Sampling and Reduced Pilot Overhead

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Orthogonal Frequency Division Multiplexing

Basic principle of OFDM

- transmit parallel data streams using several subcarriers
- design mutually orthogonal subcarriers which can be perfectly separated by the receiver
- establish orthogonality at the transmitter:
  discrete Fourier transform (DFT) serves as modulation
- maintain orthogonality after transmission:
  add a guard period between consecutive OFDM symbols
Orthogonal Frequency Division Multiplexing

- OFDM Baseband System

\[ X[n, k] \xrightarrow{S/P} \text{IFFT} \xrightarrow{P/S} D/A \xrightarrow{h(t, \tau)} A/D \xrightarrow{S/P} Y[n, k] \]

- multiplicative input-output relation after demodulation (FFT)

\[ Y[n, k] = H[n, k] X[n, k] + Z[n, k] \]

- benefits
  - high spectral efficiency
  - efficient implementation using FFT methods
  - simple one-tap equalization
Orthogonal Frequency Division Multiple Access

- subcarriers/groups of subcarriers/tiles are assigned to each user
- maintains orthogonality also in the uplink
Channel Estimation - Uplink Case

- each user transmits its data over a different channel
- base station estimates each user’s channel for coherent detection
- multiplex training data into each user’s transmit signal

→ pilot-aided channel estimation
Motivation

- accurate channel state information (CSI) for coherent detection
- flexible, adaptive resource allocation and pilot arrangement
- reduced pilot overhead
- state of the art – estimation on a per-tile basis:
  - LS-based interpolation
  - 1D and 2D MMSE-based estimators

cf. pilot arrangement in WiMAX 802.16a,e
OFDMA Uplink Model

- consider packet with $K$ subcarriers & $N$ OFDM symbols
- $U$ active users transmit data on several dedicated tiles
- each tile may contain pilot symbols
  $\rightarrow P$ pilots irregularly distributed within a packet
- noisy linear relation of receive symbols $Y[n, k]$ and $X_u[n, k]$:

$$Y[n, k] = \sum_{u=1}^{U} H_{u}[n, k] X_{u}[n, k] + Z[n, k]$$
System Model

Channel Model

- consider spreading function $S_u[m, l]$ with
  - support $[0, M_\tau - 1] \times [-\frac{M_\nu}{2}, \frac{M_\nu}{2}]$
  - delay spread $M_\tau \ll K$, Doppler spread $M_\nu \ll N$

- related to channel coefficients $H_u[n, k]$ by 2D-FFT:

$$H_u[n, k] = \frac{1}{\sqrt{KN}} \sum_{m=0}^{M_\tau - 1} \sum_{l=-\frac{M_\nu}{2}}^{\frac{M_\nu}{2}} S_u[m, l] e^{-j2\pi \left( \frac{mk}{K} - \frac{ln}{N} \right)}$$

- allows interpretation of $H_u[n, k]$ as
  - 2-D lowpass function
  - 2-D trigonometric polynomial of degree $M_\tau \times (M_\nu + 1)$
Proposed Channel Estimator

General Idea

- separate channel estimation for each user (index \( u \) omitted)
- estimate CSI of each user over the whole OFDM packet (not only on a per-tile basis)
- view channel estimation as 2-D nonuniform least-squares (LS) reconstruction problem due to irregularly distributed pilots
- adapted version of ABC algorithm (Gröchenig & Strohmer, 2001)
- properties:
  - allows for irregular distribution of samples/pilots
  - computational complexity does not scale with \( P \)
  - no second-order statistics required
Proposed Channel Estimator

Description of Method

- calculate noisy pre-estimates at pilot positions:
  \[ \hat{H}_{\text{pre}}[n_p, k_p] = \frac{Y[n_p, k_p]}{X[n_p, k_p]} \]

- LS-fit of a 2-D trigonometric polynomial of degree \( M_T \times (M_N + 1) \)

- leads to Toeplitz system with \( P \times M_T(M_N + 1) \)-matrix \( V \):
  \[ V^H V \hat{s} = V^H \hat{h}_{\text{pre}} \]
  \[ [V]_{p,q} \triangleq \frac{1}{\sqrt{KN}} e^{-j2\pi \left( \frac{m k p}{K} - \frac{l n p}{N} \right)} \]

- solve for \( \hat{s} \) (i.e., \( \hat{S}[m, l] \)) using conjugate gradient (CG)

- calculate \( \hat{H}[n, k] \)
Proposed Channel Estimator

Algorithm Summary

1. offline pre-processing:
   - compute Toeplitz matrix $\mathbf{T} = \mathbf{V}^H \mathbf{V}$ directly via 2D-FFT
   - store $\mathbf{T}$ for fixed pilot arrangement

2. online pre-processing:
   - calculate $\hat{\mathbf{h}}_{\text{pre}}$
   - obtain $\mathbf{V}^H \hat{\mathbf{h}}_{\text{pre}}$ directly via 2D-FFT

3. CG iteration:
   - calculate approximate solution $\hat{s}_r$
   - fast matrix-vector multiplication via 2D-FFT due to Toeplitz structure of $\mathbf{T}$

4. post-processing: determine $\hat{\mathbf{h}}_r$ from $\hat{s}_r$ via 2D-FFT

5. go to step 3 until $\sum_{p=1}^P |\hat{H}_r[n_p, k_p] - \hat{H}_{\text{pre}}[n_p, k_p]|^2 \leq \frac{\sigma_Z^2}{\sigma_X^2} P$
Discussion

- \( P \geq M_T(M_V + 1) \) required

- pilot arrangement, \( M_T \), and \( M_V \) play a critical role:
  \( \rightarrow \) influence condition number of Toeplitz matrix \( T \)

- CG convergence & channel estimation error depend on condition number of Toeplitz matrix \( T \)

- regularization is achieved via early termination of CG iterations

- computational complexity:
  - low complexity implementation based on FFTs
  - \( \mathcal{O}(M_T M_V \log(M_T M_V)) \) operations per iteration
Simulation Setup

- **coded OFDMA uplink system:**
  - system bandwidth \( B = 5 \text{ MHz} \), carrier frequency \( f_C = 2 \text{ GHz} \)
  - \( K = 512 \) subcarriers, \( N = 30 \) OFDM symbols/packet
  - tile size \( 3 \times 4 \) symbols, at most one pilot per tile
  - rate-1/2 convolutional code

- **channel:**
  - doubly dispersive WSSUS Rayleigh fading channel
  - uniform delay & Doppler profile
  - terminal velocity of 100 km/h \( (M_\nu = 2) \)

- **receiver:**
  - ZF equalizer
  - Viterbi decoder
Simulation Setup

- irregular pilot arrangement due to flexible resource allocation
- at most one pilot per tile
Simulation Results

BER/MSE versus SNR (9 Users, 120 Pilots/User, $M_T = 7$)

- proposed method close to ideal performance
- conventional LS estimator unable to track channel variations
- allocating more tiles while keeping $P$ fixed does not degrade performance
Simulation Results

BER/MSE versus Delay Spread (SNR = 20 dB)

- BER/MSE advantage increases with $P$
- MSE degrades only gradually for increasing delay spread
- tradeoff: delay diversity $\Leftrightarrow$ channel estimation accuracy
Conclusions

- 2-D channel estimation scheme based on irregular sampling
- Efficient implementation using CG iterations and FFTs
- Highly flexible user allocation and pilot arrangement
- Significant reduction of pilot overhead
- Excellent performance