LEAKAGE-BASED MULTICAST TRANSMIT BEAMFORMING

Multicast Interference Channel

Schematic Diagram

System Model

Input-output relationship of UE \( k_j \)

\[
y_{kj} = H_{kj}^{(j)} f_j s_j + \sum_{\ell=1,\ell \neq j}^{J} H_{kj}^{(\ell)} f_{\ell} s_{\ell} + n_{kj}
\]

Channel between UE \( k_j \) and BS \( \ell \):

\( H_{kj}^{(\ell)} \in \mathbb{C}^{M \times N} \)

Beamformer applied by BS \( j \):

\( f_j \in \mathbb{C}^{N \times 1} \)

Multicast transmit symbol of BS \( j \):

\( s_j \in \mathbb{C} \)

Additive complex-Gaussian receiver noise

\( n_{kj} \in \mathbb{C}^{N \times 1} \sim \mathcal{CN}(0, \sigma_n^2 I_M) \)

Symmetric scenario

\( M, N, J \) equal for all BSs and UEs

Receive Antenna Combining

Users apply linear receive filters:

\[
g_{kj} f_j s_j + \sum_{\ell=1,\ell \neq j}^{J} g_{kj}^{(\ell)} s_{\ell} + g_{kj}^{H} n_{kj}
\]

Effective MISO vector channel

\[
h_{kj}^{(j)} = (H_{kj}^{(j)})^H g_{kj} \in \mathbb{C}^{N \times 1}
\]

\[
s_j = (h_{kj}^{(j)})^H f_j s_j + \sum_{\ell=1,\ell \neq j}^{J} (h_{kj}^{(\ell)})^H f_{\ell} s_{\ell} + g_{kj}^{H} n_{kj}
\]

Achievable Multicast Rate

Assume single-user detection and Gaussian signalling

Achievable rate of UE \( k_j \)

\[
R_{kj} = \log_2 \left( 1 + \frac{\left| (h_{kj}^{(j)})^H f_j \right|^2}{\|g_{kj}\|^2 \sigma_n^2 + \sum_{\ell=1,\ell \neq j}^{J} \left| (h_{kj}^{(\ell)})^H f_{\ell} \right|^2} \right)
\]

Achievable multicast rate of BS \( j \)

\[
R_j = \min_{k \in \{1, \ldots, K\}} R_{kj}
\]

Goals and Contributions

Goal: maximization of achievable multicast rates

Beamformer and antenna combiner designs:

- Independent beamformer optimization through relaxation of the joint optimization problem for fixed receive antenna combiners
- “Blind” a-priori selection of antenna combiners
- Alternating optimization between beamformers and receive antenna combiners

http://www.nt.tuwien.ac.at

1/4
Leakage-Based Multicast Beamforming

Consider the following interference upper bound
\[
\sum_{\ell=1, \ell \neq j}^{J} \left| \left( \mathbf{h}_{\ell j}^{(j)} \right)^{H} \mathbf{f}_{\ell} \right|^{2} \leq \sum_{\ell=1, \ell \neq j}^{J} \alpha_{\ell j}^{(j)} \sigma_{n}^{2} =: \left( \hat{R}_{kj} \right)_{\ell \neq j}
\]

The corresponding rate lower bound is
\[
R_{kj} \geq \log_{2} \left( 1 + \frac{\left| \left( \mathbf{h}_{kj}^{(j)} \right)^{H} \mathbf{f}_{j} \right|^{2}}{1 + \alpha_{kj} \sigma_{n}^{2}} \right) =: \hat{R}_{kj}
\]

Decoupling of optimization problems
\[
\begin{align*}
\max_{\mathbf{S}_{j} \in \mathbb{C}^{N \times N}, \mathbf{f}_{j}^{(j)}} \quad & \min_{\mathbf{h}_{\ell j}^{(j)}, \alpha_{\ell j}^{(j)}, \sigma_{n}^{2}} \frac{1}{1 + \alpha_{kj}} \left( \mathbf{h}_{\ell j}^{(j)} \right)^{H} \mathbf{S}_{j} \mathbf{h}_{\ell j}^{(j)} \\
\text{subject to:} & \\
\mathbf{S}_{j} & = \mathbf{f}_{j}^{(j)} \mathbf{f}_{j}^{(j)H}, \quad \text{trace} (\mathbf{S}_{j}) \leq P_{j}, \quad \mathbf{S}_{j} \succeq 0, \quad \text{rank} (\mathbf{S}_{j}) = 1 \quad (j \neq \ell)
\end{align*}
\]

Channel covariance matrices
\[
\mathbf{R}_{\ell j}^{(j)} = \mathbf{h}_{\ell j}^{(j)} \mathbf{h}_{\ell j}^{(j)H}
\]

Similar optimization problems occur in cognitive underlay MISO multicasting

Beamformer randomization to translate optimal \( \mathbf{S}_{j}^{*} \)

to a feasible solution of \( \mathbf{f}_{j}^{(j)} \)

System Model

Apply a rank relaxation to obtain a convex semidefinite program [3]
\[
\begin{align*}
\max_{\mathbf{s}_{j} \in \mathbb{C}^{N \times N}, z \in \mathbb{R}} \quad & z \\
\text{subject to:} & \\
z & \leq \frac{1}{\alpha_{kj}} \text{trace} (\mathbf{R}_{\ell j}^{(j)} \mathbf{s}_{j}), \quad \forall k \in \{1, \ldots, K\} \\
\alpha_{kj} & = 1 + \alpha_{kj} \quad \text{trace} (\mathbf{s}_{j}) \leq P_{j}, \quad \mathbf{s}_{j} \succeq 0 \\
\text{trace} (\mathbf{R}_{\ell j}^{(j)} \mathbf{s}_{j}) & \leq \alpha_{\ell j} \sigma_{n}^{2}, \quad \forall \ell, \forall \ell, \ell \neq j
\end{align*}
\]

References

Receive Antenna Combining

- SINR-optimal for given transmit covariances $S_j$, $\forall j$
- Interference-aware MMSE filter
  $$g_{kj}^{(\text{max SINR})} = v_{\text{max}}(T_{kj})$$

$$T_{kj} = \left( \sigma_n^2 I_M + \sum_{\ell=1, \ell \neq j}^J H_{kj}^{(\ell)} S_{\ell} \left( H_{kj}^{(\ell)} \right)^H \right)^{-1} H_{kj}^{(j)} S_j \left( H_{kj}^{(j)} \right)^H$$

Causality problem:
- Beamformer calculation requires fixed receive antenna combiners and vice versa

Blind CSI Feedback with Interference-Aware Detection

1. Blind CSI feedback
   $$S_{k}^{(\text{CSI})} = c_t I_N$$
   $$c_t = \frac{\alpha_{kj}^{(t)}}{\sigma_n^2 \sigma_{kj}^{2}} \text{trace} \left( H_{kj}^{(t)} \left( H_{kj}^{(t)} \right)^H \right)$$

   $$\Rightarrow g_{kj}^{(t,\text{CSI})} \Rightarrow h_{kj}^{(t,\text{CSI})}$$

2. Beamformer calculation
   Based on $h_{kj}^{(t,\text{CSI})}$ BSs calculate beamformers
   Local/global randomization

3. Interference-Aware Detection
   Interference-aware MMSE reception based on actually applied beamformers

Full CSI Feedback with Alternating Optimization

1. Full CSI feedback
   Each UE feeds back all channel matrices
   $H_{kj}^{(t)}$ \( \forall k \) \( \forall j \) \( \forall \ell \)

2. Alternating Optimization
   1. Given $S_{k}^{(t-1)}$ BSs calculate MMSE filters
   2. BSs exchange effective channels $h_{kj}^{(t,\ell)}$
   3. BSs update transmit covariances $S_{k}^{(t)}$
   4. BSs exchange transmit covariances

3. Interference-Aware Detection
   Interference-aware MMSE reception based on actually applied beamformers

Advantages/Disadvantages

Blind approach:
- Lower complexity
- Reduced feedback and backhaul overhead
- Worse performance

Full CSI approach:
- Better performance
- Large feedback and backhaul overhead
- High complexity
- Convergence not yet established
  (finite number of iterations applied)
### Numerical Experiment

Simulation parameters:
- \( J = 3 \) BSs each equipped with \( N = 8 \) antennas
- \( K = 2/4/8 \) UEs with \( M = 2 \) antennas
- Rayleigh fading channels
- Serving BS received on average with 0 dBW
- Interfering BSs received on average with 3 dBW
  \( \rightarrow \) average SIR = 0 dB
- Same leakage parameter for all UEs
  \( \alpha_{it}^{(j)} = \alpha, \forall i, t, j \)
- not Pareto-optimal

### Considered Transceivers

RLBM blind (max EV/local/global):
- Blind CSI feedback
- max EV: beamforming along maximum EV
- local/global randomization

RLBM alternating:
- Full CSI feedback
- Alternating optimization with global rand.

MaxMin (time sharing):
- MaxMin SNR beamforming


### Open Issues

- How to select the leakage parameters to achieve Pareto-optimality?
- Is independent optimization based on the SINR lower bound equivalent to joint optimization?
- How large is the optimality gap caused by the rank relaxation?
- How to establish convergence of alternating optimization between beamformers and receivers?

### Achievable Rate Simulations

**K = 2 UEs per BS**

- RLBM alternating
- RLBM blind
- MaxMin (time sharing)

**K = 4 UEs per BS**

- RLBM alternating
- RLBM blind (global)
- RLBM blind (local)
- RLBM blind (max EV)
- MaxMin (time sharing)

**K = 8 UEs per BS**

- RLBM alternating
- RLBM blind (global)
- RLBM blind (local)
- RLBM blind (max EV)
- MaxMin (time sharing)