Abstract—An improved SINR metric is proposed for the random beamforming scheme introduced by Sharif and Hassibi, when the channel observation used to compute the SINR is known to be noisy or outdated. The effect of noise on the MIMO channel estimate is accounted for using results on the perturbation of the eigenspaces of Hermitian matrices. The new metric, designed as a conservative estimate of the real SINR, is based on expectations of bounds on the signal and interference power. It is shown through simulations that it can noticeably reduce the outage probability, in the realistic setting of a $4 \times 2$ antennas system, at the cost of a minor reduction of the achievable sum-rate.

I. INTRODUCTION

Random beamforming [1] is a transmission technique suitable for the broadcast channel with limited channel state information at the transmitter (CSIT). By serving the users with the best channels, it is able to exploit multiuser diversity while requiring a fairly low amount of feedback about the channel state (in the form of the Signal-to-Interference plus Noise Ratio - SINR). However, SINR estimation can be inaccurate due to channel estimation noise and temporal variation of the channel, which can generate outages if adaptive coding and modulation are used. Therefore, it is desirable to investigate the reliability of the measured SINR, in order to enable better scheduling decisions.

This problem has been addressed in the case of a single antenna at the receiver, in [2] where a SINR back-off scheme is proposed, and in [3] and [4] with a rate back-off mechanism. However, when multiple antennas are present at the receiver, analysis of the perturbation of the measured channel can provide useful information about the true SINR, as demonstrated in the present article. More specifically, the object of this paper is to investigate the mismatch between the measured SINR and the real SINR in the case of multiple receive antennas, and to provide a conservative estimate of the true SINR based on the knowledge of the measured channel and the variance of the estimation error. We show by simulation that the use of this metric instead of the measured SINR has the potential to noticeably reduce the amount of channel outages.

We note that the topic of SINR perturbation has been studied also in [5]. However, in [5], a perturbation of the beamforming vectors is considered (their orthogonality is relaxed into $\epsilon$-orthogonality), whereas we focus on perturbations of the channel itself. Note also that although the channel estimation error can have multiple causes, namely estimation error (which can be combated by increasing the power dedicated to training) or variation of the channel between the training phase and the data transmission phase (which can be mitigated by reducing the time interval between the two). However, in the present article, we do not attempt to distinguish estimation error and actual channel variation, and rather choose to model the difference as additive Gaussian noise.

This article is organized as follows: the system model is introduced in section II, the SINR lower bound is established in section III, together with its expectation. The influence of the proposed channel quality metric is studied in section IV by ways of simulation. Finally, section V concludes the paper.

II. SYSTEM MODEL

Let us consider the downlink of a wireless communications systems. A transmitter is equipped with $M$ antennas and $K$ users with $N$ antennas each. The baseband model for user $k$ is given by

$$ y_k(t) = H_k(t)x(t) + n_k(t), $$

where $x(t)$ is an $M$ dimensional vector representing the transmitted signal, $H_k(t)$ is an $N \times M$ matrix representing the MIMO channel (assumed frequency-flat) experienced by user number $k$ at time $t$, and $n_k(t)$ denotes the noise experienced by user $k$. Here, the noise is modeled as Gaussian i.i.d. with $\mathbb{E}[n_k(t)n_k(t)^H] = I$. Note that we assume that all users have the same number $N \geq 2$ of antennas purely for notational simplicity, and without loss of generality.

Let $H_k$ denote the channel estimated by this user during the SINR calculation phase of random beamforming. As noted before, in general, $H_k \neq H_k$, due to both estimation noise and time variation of the channel. Let $E_k = H_k - H_k$ denote the channel measurement error, and assume that it is a Gaussian random matrix with i.i.d. components of variance $\sigma^2_E$.

The random beamforming method [1] operates as follows: the base station randomly picks a set of $M$ orthogonal beamforming vectors $w_1, \ldots, w_M$. Let $W = [w_1 \ldots w_M]$ denote the unitary matrix of beamforming vectors. A number of independently encoded data streams are transmitted, each stream being associated to one of the beamforming vectors. Although not generally optimal in the case of a Gaussian vector broadcast channel (see [6]), this approach is of interest since the amount of channel knowledge required to make the scheduling decision is lower than the full channel knowledge required by optimal approaches, thus saving feedback bandwidth. In general, granting channel access to the best (in
the highest achievable mutual information sense) users will maximize the sum-throughput of the system. More involved selection criteria involving e.g. fairness between users [7] or taking delay constraints into account are sometimes desirable, and require relaxing the sum-rate maximization requirement. In the present paper, we will focus on the case where the SINR is used as the channel quality metric.

Let us further analyze the random beamforming mechanism, and let \( \mathbf{s}(t) \) denote the vector containing the \( M \) symbols associated to each stream and transmitted at time \( t \). The signal received by user \( k \) is therefore a superposition of all \( M \) streams, with weights depending on the beamforming vectors and their own channel, as

\[
y_k = \sum_{i=1}^{M} H_k w_i s_i + n_k = H_k \mathbf{w}^H \mathbf{s} + n_k. \tag{2}
\]

The goal of the training phase in the random beamforming method is to associate each beamforming vector to a user (here we assume that the number of users, denoted by \( K \), is at least \( M \)). Assuming that all beams are always in use, and that the power budget is \( \rho = \text{Tr} [\mathbb{E} \{ \mathbf{s} \mathbf{s}^H \} ] \), the SINR of stream \( b \) as experienced by user \( k \) can be written

\[
\text{SINR}_{k,b} = \frac{\mathbf{w}_b^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_b}{\sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_i + (M/\rho)}. \tag{3}
\]

Assuming (without loss of generality) that \( b = 1 \), and denoting \( \mathbf{W}_\perp = [\mathbf{w}_2 \ldots \mathbf{w}_M] \), this becomes

\[
\text{SINR}_{k,1} = \frac{\mathbf{w}_1^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_1}{\text{Tr} (\mathbf{W}_\perp \mathbf{H}_k^H \mathbf{H}_k \mathbf{W}_\perp) + (M/\rho)}. \tag{4}
\]

Note that if several streams can be assigned to the same user, SINR analysis based on the above formula is not sufficient, since joint decoding of the streams could effectively suppress part of the interference, depending on the receiver structure. However, as noted in [8], the probability of several beams being assigned to one user vanishes for large number of users, and we will therefore neglect this case.

We will now focus on the perturbation of this expression when \( \mathbf{H} \) is used instead of \( \mathbf{H} \) in the above formula.

### III. SINR Perturbation

As already hinted in Sharif and Hassibi’s paper [1], for a sufficiently large number of transmit antennas, the SINR in random beamforming systems is dominated by interference, i.e., proper matching of a beam to the channel eigenmode (which enables interference nulling at the receiver) is more important than the actual received power alone. Therefore, our perturbation analysis will be based on the eigendecomposition of the Hermitian matrix \( \mathbf{A} = \mathbf{H}^H \mathbf{H} \), i.e., \( \mathbf{A} = \mathbf{V} \mathbf{D} \mathbf{V}^H \) where \( \mathbf{V} \) is the unitary matrix containing the eigenvectors, and \( \mathbf{D} \) is the diagonal matrix containing the (real) eigenvalues of \( \mathbf{A} \).

The perturbation analysis that we propose in this section (and the stochastic version that follows) is largely inspired by the work of Stewart in [9]. However, the analysis presented here is done for the particular case where both the original and the perturbed matrices are Hermitian, which is not the case in the original analysis.

Let us consider the perturbed matrix \( \mathbf{\tilde{A}} = \mathbf{\tilde{H}}^H \mathbf{\tilde{H}} = \mathbf{A} + \mathbf{B} \) where \( \mathbf{B} = \mathbf{E}^H \mathbf{H} + \mathbf{H}^H \mathbf{E} + \mathbf{E}^H \mathbf{E} \). We will assume that \( \mathbf{A} \) is used during the training phase to estimate the SINR. We consider in particular its eigendecomposition \( \mathbf{\tilde{A}} = \mathbf{V} \mathbf{D} \mathbf{V}^H \).

For simplicity, we investigate separately the perturbation of the numerator and denominator of the SINR formula (3). Since \( \mathbf{V} \) is unitary, one can always write the following expression for the power of the signal of interest (numerator)

\[
P = \mathbf{w}_1^H \mathbf{A} \mathbf{w}_1 = \mathbf{w}_1^H \mathbf{\tilde{V}} \mathbf{\tilde{V}}^H \mathbf{V} \mathbf{D} \mathbf{V}^H \mathbf{\tilde{V}}^H \mathbf{w}_1, \tag{5}
\]

and the following expression for the interference power

\[
I = \text{Tr} [\mathbf{W}_\perp \mathbf{A} \mathbf{W}_\perp] = \text{Tr} \left[ \mathbf{W}_\perp \mathbf{\tilde{V}} \mathbf{\tilde{V}}^H \mathbf{V} \mathbf{D} \mathbf{V}^H \mathbf{\tilde{V}}^H \mathbf{W}_\perp \right]. \tag{6}
\]

Interestingly, different terms can be isolated above: the terms denoted by \(^1\) in (5) and (6) represent the mismatch between the beamforming vector of the first stream \( \mathbf{w}_1 \) and the eigenvectors \( \mathbf{V} \) of the measured channel matrix \( \mathbf{H} \), and the mismatch between the beamforming vectors of the interfering streams and \( \mathbf{\tilde{V}} \) respectively. We denote

\[
\mathbf{\tilde{V}}^H \mathbf{w}_1 = \begin{bmatrix} f_1 \\ f_\perp \end{bmatrix} \quad \text{and} \quad \mathbf{\tilde{V}}^H \mathbf{W}_\perp = \begin{bmatrix} g_1 \\ G_{\perp} \end{bmatrix}. \tag{7}
\]

\( f_1 \) is a scalar, \( f_\perp \) is a \((M-1) \times 1\) vector, \( g_1 \) is a \(1 \times (M-1)\) vector and \( G_{\perp} \) is a \((M-1) \times (M-1)\) matrix. For an SINR metric based on the knowledge of the measured channel \( \mathbf{H} \), all terms in (7) can be measured during the training phase and \( \mathbf{D} \) can be approximated by \( \mathbf{\tilde{D}} \). It remains to investigate the terms denoted by \(^2\) in (5) and (6) which represent the subspace estimation error (indeed, if \( \mathbf{E} = 0 \), \( \mathbf{V}^H \mathbf{\tilde{V}} \) becomes the identity matrix).

The bound presented below is based on the analysis of the effects of \( \mathbf{E} \) on the product \( \mathbf{V}^H \mathbf{\tilde{V}} \). Let us denote \( \mathbf{V} = [\mathbf{v}_1 \mathbf{V}_\perp] \) where we separate the first eigenvector from the others and \( \mathbf{D} = \text{diag} \{ \lambda_1, \Lambda_{\perp} \} \), \( \lambda_1 \) is the \(1 \times (M-1)\) vector \( \{ \lambda_2, \ldots, \lambda_M \} \). Similarly, \( \mathbf{\tilde{V}} = [\mathbf{\tilde{v}}_1 \mathbf{V}_\perp] \) and \( \mathbf{\tilde{D}} = \text{diag} \{ \tilde{\lambda}_1, \tilde{\Lambda}_{\perp} \} \). We will be concerned with the effect of \( \mathbf{B} \) to the subspace of \( \mathbf{A} \) spanned by \( \mathbf{v}_1 \) which we denote by \( \mathcal{S}(\mathbf{v}_1) \). The subspace spanned by \( \mathbf{v}_1 \) approaches \( \mathcal{S}(\mathbf{v}_1) \) as \( \mathbf{B} \) approaches zero. Let us now characterize this perturbation more precisely: following [9], we consider the perturbation of the first invariant subspace of \( \mathbf{A} \), through the first-order expansion of \( \mathbf{\tilde{v}}_1 \):

\[
\mathbf{\tilde{v}}_1 = \mathbf{v}_1 + \mathbf{V}_\perp \mathbf{p}, \tag{8}
\]

where \( \mathbf{p} \) is a \((M-1) \times 1\) weights vector which is presumed small. \( \mathbf{\tilde{A}} \) basis for the orthogonal complement of \( \mathcal{S}(\mathbf{v}_1) \) is then
given by the columns of
\[ \hat{V}_\perp = V_\perp - \gamma \lambda_i^p H^T. \]  
(9)

These properties can be summarized as \( \bar{V}_\perp^H (A + B) \bar{V}_\perp = 0 \), or
\[ (V_\perp + V_\perp p)^H (A + B) (V_\perp - \gamma \lambda_i^p H^T) = 0. \]  
(10)

Solving (10) for \( p \), we will find a measure of the distance between \( S(\gamma_i) \) and \( S(\gamma_i) \), i.e., the perturbation of the eigenvalue \( \lambda_i \). Let us define
\[ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} \\ \end{bmatrix} = \begin{bmatrix} \frac{\bar{V}_\perp}{\bar{V}_\perp} \\ \end{bmatrix} \begin{bmatrix} \gamma_i & V_\perp \\ \end{bmatrix}. \]  
(11)

Using these notations, (10) can be rewritten as
\[ \begin{bmatrix} b_{11} + \lambda_i p + B_{22} p - p \lambda_i - p b_{11} - p b_{21} \end{bmatrix} = 0, \]  
(12)

with \( \lambda_i = \text{diag}(\gamma_i) \). If we drop the second-order terms \( (B_{22} p, b_{11} \) and \( b_{21} p \)) from (12), we get the first order perturbation equation
\[ \lambda_i \bar{p} - \bar{p} \lambda_i = -b_{21}, \]  
(13)

where \( \bar{p} \) is an approximation of \( p \). Defining the linear operator \( T \) as \( T(p) = \lambda_i p - p \lambda_i \), eq. (12) becomes \( b_{21} = -T(\bar{p}) \).

Let us define
\[ \delta = \inf_{\|p\|_F = 1} \|T(p)\|_F, \]  
(14)

where \( \| \cdot \|_F \) denotes the Frobenius norm. According to [9, Theorem 4.3], \( \delta \) is a function of the eigenvalues of \( A \): \( \delta = \min \{|\lambda_1 - \lambda_i| : \lambda_i \in \Lambda_i\} \). The bound on the perturbation is a consequence of the definition of \( \delta \):
\[ \delta \|\bar{p}\|_F \leq \|T(\bar{p})\|_F = \|b_{21}\|_F. \]  
(15)

A. Stochastic bound

Since the above bound depends on \( b_{21} \), which is itself a function of the unknown noise \( E \), we have to propose a stochastic version of the bound. Let us recall the definition of the stochastic bound (again from [9]):
\[ \|E\|^2_F = \mathbb{E} \|E\|^2_F. \]  
(16)

Let us examine in further detail the structure of \( b_{21} \): (11) yields
\[ b_{21} = V_\perp H H^T E_\perp^1 \]  
(17)

Dropping the second order term \( V_\perp H E_\perp^1 \) and writing the channel matrix \( H \) according to its singular value decomposition \( H = UD^2 V^H \) leads to
\[ b_{21} = V_\perp H E_\perp^1 UD^2 V^H \]  
(18)

where \( E' = UH E \) has the same Gaussian i.i.d. distribution as \( E \) since \( U \) is unitary. Note that the two terms in the above expression are statistically independent, since the first depends only on the first row of \( E' \), and the second on the remaining rows. Therefore, their stochastic norms simply add up. Direct application of [9, Theorem 2.5] yields
\[ \|b_{21}\|_F^2 = \|V_\perp^H \gamma_i \lambda_i^2\|^2 E + \sum_{i=2}^M \lambda_i \|\gamma_i\|^2 F^2 E \]  
(21)

This yields the stochastic version of the bound of eq. (15):
\[ \|\bar{p}\|_F \leq \delta^{-1} \sqrt{(M - 1) \lambda_1 + \text{Tr}(\Lambda_i)} \sigma_E. \]  
(22)

B. Perturbation of the SINR

In the following we propose a stochastic SINR metric based on the knowledge of the measured channel \( H \) and the variance \( \sigma_E^2 \). For tractability, we investigate here the use of \( \|w_h\|_F^2 M \) as the SINR metric, where \( w_h \leq \mathbb{E}[w_h^H H^H w_h] \) and \( f_{\text{ub}} \geq \mathbb{E}[\text{Tr}(W^H H^H W)] \). Those bounds are obtained by first bounding the power terms with functions of \( \|\bar{p}\|_F \), and then using the perturbation bound (22) since \( E \) (and therefore \( p \)) is not known deterministically.

Let us consider the numerator of the SINR expressed in (5). Using the fact that the subspace estimation error denoted by the term \( q^2 \) in (5) can be written as
\[ V_\perp^H \bar{V} = \begin{bmatrix} 1 & -p^H \bigg| \mathbb{I}_{M-1} \end{bmatrix}, \]  
(23)

we obtain the following expression for the received signal power
\[ \begin{aligned} P &= \mathbb{E}^H A \mathbb{E}^1 = \lambda_1 \|f_1 - p^H f_\perp\|^2 \\
&+ \|p f_1 + f_\perp^H A_{\perp} (p f_1 + f_\perp) \|^2 \\
&= \lambda_1 \|f_1\|^2 - \sum_{i=1}^{M-1} 2 \mathbb{E}[f_i f_1^H f_\perp (\lambda_1 - \lambda_\perp (i))] \\
&+ \lambda_1 \sum_{i=1}^{M-1} |f_1^H f_\perp (i) p_i |^2 + |f_1| \sum_{i=1}^{M-1} |p_i|^2 \lambda_\perp (i) \\
&+ \sum_{i=1}^{M-1} |f_\perp (i) p_i|^2 \lambda_\perp (i), \end{aligned} \]  
(24)

with \( p_i \) and \( f_\perp (i) \) the \( i \)-th component of \( p \) and \( f_\perp \) respectively, \( \lambda_\perp (i) = \lambda_i + 1 \) and \( \mathbb{H} \) the real part operator.

A lower bound is obtained using the fact that \( \mathbb{R}(f_i f_1^H f_\perp (\lambda_1 - \lambda_\perp (i))) \leq |f_1| |p_i| |f_\perp (i) (\lambda_1 - \lambda_\perp (i))| \).

Then, applying the Jensen’s inequality (with \( \mathbb{E}[f_i f_1^H f_\perp (\lambda_1 - \lambda_\perp (i))]) \) we obtain
\[ \sum_{i=1}^{M-1} |p_i|^2 \xi_i \leq \sum_{i=1}^{M-1} |p_i|^2 \xi_i \sum_{i=1}^{M-1} |p_i|^2 \]  
(25)

This yields \( \sum_{i=1}^{M-1} 2 \mathbb{E}[f_i f_1^H f_\perp (\lambda_1 - \lambda_\perp (i))] \leq \gamma_1 \|p\|_F^2 \), where \( \gamma_1 = 2 |f_1| \sqrt{\sum_i |\xi_i|} \max_i \xi_i \), and therefore
\[ P \geq \lambda_1 \|f_1\|^2 - \gamma_1 \|p\|_F + \sum_{i=1}^{M-1} |f_\perp (i) p_i|^2 \lambda_\perp (i), \]  
(27)
Taking the expectation (over $E$) on both sides of this inequality yields

$$
\mathbb{E}[P] \geq P_{S, tb} = \lambda_1 |f_i|^2 - \gamma_1 \Delta + \sum_{i=1}^{M-1} |f_{\perp}(i)|^2 \Delta_{\perp}(i). \quad (28)
$$

We proceed similarly for the interference power term. With (7) and (23) we obtain the following expression for the interference power (6)

$$
I = \text{Tr} \left[ \left( \frac{g_1}{G_\perp} \right)^H \left[ \begin{array}{cc} 1 & -P^H \\ -P & \text{I}_{M-1} \end{array} \right] \right] \left( \begin{array}{cc} 1 & -P^H \\ -P & \text{I}_{M-1} \end{array} \right) \left( \begin{array}{c} g_1 \\ G_\perp \end{array} \right)
$$

$$
= \text{Tr} \left[ \lambda_1 \left( g_1^H g_1 - g_1^H P^H G_\perp - G_\perp^H P g_1 + G_\perp^H \text{I}_{M-1} \right) \right] + \sum_{i=1}^{M-1} \sum_{j=1}^{M-1} |g_{\perp}(i,j)|^2 \Delta_\perp(i) + \sum_{i=1}^{M-1} |g_{\perp}(i,j)|^2 \Delta_\perp(i,j).
$$

(29)

If we drop the second order terms ($G_\perp^H \text{I}_{M-1} g_1$ and $g_1^H \text{I}_{M-1} g_1$) from (29), we obtain the following expression

$$
I = \lambda_1 ||g_1||^2 + \sum_{i=1}^{M-1} 2 \Re \left\{ \rho \sum_{j=1}^{M-1} g_{\perp}^*(j) G_{\perp}(i,j) \right\} + \sum_{i=1}^{M-1} \sum_{j=1}^{M-1} |g_{\perp}(i,j)|^2 \Delta_\perp(i,j).
$$

(30)

Again, we upper bound the summation over the real part in (30) by

$$
\sum_{i=1}^{M-1} 2 \Re \left\{ \rho \sum_{j=1}^{M-1} g_{\perp}^*(j) G_{\perp}(i,j) \right\} \leq 2 \sqrt{\left( \sum_{i=1}^{M-1} \zeta_i \right) \max \|p\|_F = \gamma_2 \|p\|_F},
$$

(31)

with $\zeta_i = |\sum_{j=1}^{M-1} g_{\perp}^*(j) G_{\perp}(i,j) (\Delta_\perp(i) - \lambda_1)|$ and $\gamma_2 = 2 \sqrt{\left( \sum_{i=1}^{M-1} \zeta_i \right) \max \|p\|_F}$. Thus we get the upper bound on the interference power

$$
I \leq \lambda_1 ||g_1||^2 + \gamma_2 \|p\|_F + \sum_{i=1}^{M-1} \sum_{j=1}^{M-1} |g_{\perp}(i,j)|^2 \Delta_\perp(i). \quad (32)
$$

Taking the expectation (over $E$) on both sides of this inequality yields

$$
\mathbb{E}[I] \leq I_{S, ab} = \lambda_1 ||g_1||^2 + \gamma_2 \Delta + \sum_{i=1}^{M-1} \sum_{j=1}^{M-1} |g_{\perp}(i,j)|^2 \Delta_\perp(i).
$$

(33)

We finally obtain a conservative (likely biased) estimate of the SINR achieved over the real channel by using (33) and (28) in $P_{S, ab} = M/p$. 

### IV. Simulation Results

In this section we compare the sum-rate and outage probability of the proposed feedback scheme to those achieved by other feedback metrics. Let us first recall the scheduling algorithm used in this comparison.

Each user feeds back the highest measured SINR value and the index of the beam that leads to this highest SINR value, i.e., for user $k$, the feedback values consist of $(\text{SINR}_{k,b_{k}}, b_{k})$, where $b_{k} = \text{argmax}_{i=1,...,M} \text{SINR}_{k,i}$. Among all the users that fed back the $i$-th beamforming vector the scheduler selects the user $k_i$ that leads to the highest SINR value for this particular beam, i.e., the use of beam $i$ is granted to user $k_i = \text{argmax}_{i} (\text{SINR}_{k,b_i})$ for $b_i = i$. Data transmitted on beam $i$ is encoded with rate $R_i = \log_2 (1 + \text{SINR}_{k_i})$ bits/s/Hz. The transmission is in outage if $R_i > \log_2 (1 + \text{SINR}_{\text{inst},i})$ with SINR$_{\text{inst},i}$ the true SINR value of the user $k_i$ for beam $i$.

We compare the following methods of SINR estimation:
- the perfect SINR feedback, where $H = \hat{H},$
- the measured SINR feedback (denoted by SINR-Meas), where the SINR is evaluated by directly plugging $H$ into the SINR formula (3),
- the SINR Expected Lower Bound (SINR-ELB) introduced in section III,
- the SINR back-off scheme (SINR-BO) [2] applied to the measured SINR. The optimal back-off coefficient is evaluated numerically for each value of $\sigma_E^2$.

For all simulations, the signal-to-noise ratio is $\rho = 10\, \text{dB}$, and the number of user is set to $K = 64$, and $H$ is Gaussian i.i.d. (Note however that the results in the previous section are derived for a deterministic $H$). The sum-rate and the outage probability $p_{\text{out}}$ are plotted versus the relative channel measurement error denoted by $\sigma_H^2/\sigma_E^2$. Performance of a $2 \times 2$ antennas systems is depicted on Fig. 1 and 2, whereas Figs. 3 and 4 correspond to $4 \times 2$ antennas systems.

For both the $2 \times 2$ and $4 \times 2$ antennas cases, the scheme that has perfect SINR information is clearly superior to the others. The SINR-ELB performs better than the SINR-Meas feedback over the complete range of SINR values, due to the high outage probability incurred by latter. Conversely, the SINR-ELB metric ensures that the transmit data rate assigned to the scheduled users are chosen moderately, thereby reducing channel outages. In $2 \times 2$ systems, the SINR-BO feedback method outperforms the SINR-ELB both in terms of sum-rate and in terms of outage probability. However, in $4 \times 2$ antennas systems, the SINR-ELB metric provides a noticeable improvement in terms of outage probability, while approaching the performance of the SINR-BO for $\sigma_H^2/\sigma_E^2 > 15\, \text{dB}$. Clearly for high $\sigma_H^2/\sigma_E^2$ values all the schemes perform equally.

Therefore, the newly introduced SINR-ELB metric is successful in reducing the outage probability with a marginal loss in the sum-rate in the case of $4 \times 2$ systems. However, for $2 \times 2$ antennas systems, the simpler SINR-BO scheme is superior.

### V. Conclusion

A perturbation bound for the eigenspaces of an Hermitian matrix was derived, and applied to the evaluation of (respec-
random beamforming. In certain situations, such as the interference plus noise terms of the SINR in the context of advanced receivers capable of mitigating interference through e.g. LMMSE equalization.

Possible extensions include considering directly the first- and second-order statistics of the SINR, and considering more advanced receivers capable of mitigating interference through e.g. LMMSE equalization.

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