WELLENAUSBREITUNG
Formelsammlung

INSTITUT FÜR NACHRICHTENTECHNIK
UND HOCHFREQUENZTECHNIK

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Maxwellsche Theorie

$$\vec{\nabla} \cdot \vec{S} = \lim_{\mathcal{V} \to 0} \frac{1}{V} \oint_{\Sigma} \vec{S} \cdot \mathrm{d}\vec{F} = -\frac{\partial}{\partial t} \rho$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) \equiv 0$$

$$\vec{\nabla} \times \vec{H} = \vec{S} + \frac{\partial}{\partial t} \vec{D}$$

$$\vec{\nabla} \cdot \vec{S} = -\frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{D}) = -\frac{\partial}{\partial t} \rho$$

$$\vec{\nabla} \cdot \vec{S} + \frac{\partial}{\partial t} \rho = 0$$

$$\begin{aligned} \vec{\nabla} \cdot \vec{D} &= \rho \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial}{\partial t} \vec{B} \\ \vec{\nabla} \times \vec{H} &= \vec{S} + \frac{\partial}{\partial t} \vec{D} \end{aligned}$$

$$\begin{aligned} \int_{\Sigma} \vec{D} \cdot \mathrm{d}\vec{F} &= \int_{\tau} \rho \, \mathrm{d}V \\ \int_{\Sigma} \vec{B} \cdot \mathrm{d}\vec{F} &= 0 \\ \oint \vec{E} \cdot \mathrm{d}\vec{l} &= -\frac{\partial}{\partial t} \int_{\Sigma} \vec{B} \cdot \mathrm{d}\vec{F} \\ \oint \vec{H} \cdot \mathrm{d}\vec{l} &= \int_{\Sigma} \vec{S} \cdot \mathrm{d}\vec{F} + \frac{\partial}{\partial t} \int_{\Sigma} \vec{D} \cdot \mathrm{d}\vec{F} \end{aligned}$$

$$\vec{F} = q\Big(\vec{E} + \vec{v} \times \vec{B}\Big)$$

$$\vec{S} = \sigma \vec{E}$$

$$\vec{S} = \rho \vec{v}$$

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$$

$$\vec{B} = \mu \vec{H} = \mu_r \mu_0 \vec{H}$$

$$\frac{\partial}{\partial t} \rho(\vec{r}, t) + \frac{\sigma}{\varepsilon} \rho(\vec{r}, t) = 0$$

$$\rho(\vec{r}, t) = \rho_0(\vec{r}) e^{-\frac{\sigma}{\varepsilon} t}$$

$$\tau_{\mathrm{D}} = \frac{\varepsilon}{\sigma}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{E}(x,y,z,t) = \vec{E}(\vec{r},t) = \mathrm{Re}\{\vec{E}(\vec{r})\, e^{j\omega t}\} = \frac{1}{2}\Big(\vec{E}(\vec{r})\, e^{j\omega t} + \vec{E}^{*}(\vec{r})\, e^{-j\omega t}\Big)$$

$$\vec{\nabla} \times \vec{H} = \vec{S} + j\omega\vec{D} = \sigma\vec{E} + j\omega\varepsilon\vec{E} = j\omega\delta\vec{E}$$

$$\delta = \varepsilon + \frac{\sigma}{j\omega} = \varepsilon - j\frac{\sigma}{\omega}$$

$$\tan\theta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}$$

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= 0 \\ \vec{\nabla} \cdot \vec{H} &= 0 \\ \vec{\nabla} \times \vec{E} &= -j\omega\mu\vec{H} \\ \vec{\nabla} \times \vec{H} &= j\omega\delta\vec{E} \end{aligned}$$

$$\vec{P}(t) = \vec{E}(t) \times \vec{H}(t)$$

$$\vec{\nabla} \cdot \vec{P}(t) = -\sigma \underbrace{\vec{E}(t) \cdot \vec{E}(t)}_{\vec{E}^2(t)} - \frac{\partial}{\partial t}\Big(\frac{\varepsilon}{2} \underbrace{\vec{E}(t) \cdot \vec{E}(t)}_{\vec{E}^2(t)} + \frac{\mu}{2} \underbrace{\vec{H}(t) \cdot \vec{H}(t)}_{\vec{H}^2(t)}\Big)$$

$$w_{\mathrm{e}}(t) = \frac{\varepsilon}{2}\vec{E}^2(t)$$

$$w_{\mathrm{m}}(t) = \frac{\mu}{2}\vec{H}^2(t)$$

$$p_{\mathrm{v}}(t) = \sigma\vec{E}^2(t)$$

$$\int_{\mathcal{V}} \vec{\nabla} \cdot \vec{P}(t)\mathrm{d}V = \oint_{\Sigma} \vec{P}(t) \cdot \mathrm{d}\vec{F}$$

$$-\frac{\partial}{\partial t}\int_{\mathcal{V}} \Big(w_{\mathrm{e}}(t) + w_{\mathrm{m}}(t)\Big)\mathrm{d}V = \oint_{\Sigma} \vec{P}(t) \cdot \mathrm{d}\vec{F} + \int_{\mathcal{V}} p_{\mathrm{v}}(t)\mathrm{d}V$$

$$\vec{E}(\vec{r},t) = \frac{1}{2}\Big(\vec{E}(\vec{r})\, e^{j\omega t} + \vec{E}^{*}(\vec{r})\, e^{-j\omega t}\Big)$$

$$\vec{E}(t) \cdot \vec{E}(t) = \frac{1}{4}\Big(\vec{E}(\vec{r}) \cdot \vec{E}(\vec{r})\, e^{2j\omega t} + 2\vec{E}(\vec{r}) \cdot \vec{E}^{*}(\vec{r}) + \vec{E}^{*}(\vec{r}) \cdot \vec{E}^{*}(\vec{r})\, e^{-2j\omega t}\Big)$$

$$\overline{\vec{E}(t) \cdot \vec{E}(t)} = \frac{1}{2}|\vec{E}(t)|^2$$

$$\overline{w_{\mathrm{e}}(t)} = w_{\mathrm{e}} = \frac{\varepsilon}{4}|\vec{E}(t)|^2$$

$$\overline{w_{\mathrm{m}}(t)} = w_{\mathrm{m}} = \frac{\mu}{4}|\vec{H}(t)|^2$$

$$\overline{p_{\mathrm{v}}(t)} = p_{\mathrm{v}} = \frac{\sigma}{2}|\vec{E}(t)|^2$$

$$\vec{T} = \frac{1}{2}\vec{E} \times \vec{H}^{*} = \vec{T}_{\mathrm{w}} + j\vec{T}_{\mathrm{b}}$$

$$\begin{aligned}\oint \vec{E}\cdot \mathrm{d}\vec{l} &= E_{t1}\Delta l + E_{n1}\Delta x + E_{n2}\Delta x - E_{t2}\Delta l - E_{n2}\Delta x - E_{n1}\Delta x \\ &= (E_{t1}-E_{t2})\Delta l - 0E_{n1} + 0E_{n2} = -\frac{\partial}{\partial t}\int_F \vec{B}\cdot \mathrm{d}\vec{F} = 0\end{aligned}$$

$$E_{t1} = E_{t2}$$

$$H_{t1} = H_{t2}$$

$$\int \vec{D}\cdot \mathrm{d}\vec{F} = (D_{n1}-D_{n2})\Delta F = \rho_{\mathrm{S}}\Delta F$$

$$D_{n1} - D_{n2} = \rho_{\mathrm{S}} \quad \rightarrow \quad \varepsilon_1 E_{n1} - \varepsilon_2 E_{n2} = \rho_{\mathrm{S}}$$

$$B_{n1} = B_{n2} \quad \rightarrow \quad \mu_1 H_{n1} = \mu_2 H_{n2}$$

$$\begin{aligned}\vec{n}\cdot \vec{D}_1 &= \rho_{\mathrm{S}} \\ \vec{n}\cdot \vec{B}_1 &= 0 \\ \vec{n}\times \vec{E}_1 &= \vec{0} \\ \vec{n}\times \vec{H}_1 &= \vec{K}\end{aligned}$$

$$\begin{aligned}\vec{n}\cdot (\vec{D}_1 - \vec{D}_2) &= \rho_{\mathrm{S}} \\ \vec{n}\cdot (\vec{B}_1 - \vec{B}_2) &= 0 \\ \vec{n}\times (\vec{E}_1 - \vec{E}_2) &= \vec{0} \\ \vec{n}\times (\vec{H}_1 - \vec{H}_2) &= \vec{K}\end{aligned}$$

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\vec{E} - \mu\sigma \frac{\partial}{\partial t}\vec{E} = 0$$

$$\nabla^2 \vec{E} + (\omega^2 \mu\varepsilon - j\omega\mu\sigma)\vec{E} = 0$$

$$\nabla^2 \vec{E} + \omega^2 \mu\delta \vec{E} = 0$$

$$\nabla^2 \vec{H} + \omega^2 \mu\delta \vec{H} = 0$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$k = \omega\sqrt{\mu\delta}$$

$$\Psi(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$\frac{1}{X(x)}\frac{\partial^2}{\partial x^2}X(x) + \frac{1}{Y(y)}\frac{\partial^2}{\partial y^2}Y(y) + \frac{1}{Z(z)}\frac{\partial^2}{\partial z^2}Z(z) + \underbrace{k^2}_{\text{const.}} = 0$$

$$\frac{1}{X(x)}\frac{\partial^2}{\partial x^2}X(x) = -k_x^2$$

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2 \mu\delta$$

$$\frac{\partial^2}{\partial x^2}X(x) + k_x^2\,X(x) = 0$$

$$
\begin{aligned}
\frac{\partial}{\partial y}H_z - \frac{\partial}{\partial z}H_y &= j\omega\delta E_x \\
\frac{\partial}{\partial z}H_x - \frac{\partial}{\partial x}H_z &= j\omega\delta E_y \\
\frac{\partial}{\partial x}H_y - \frac{\partial}{\partial y}H_x &= j\omega\delta E_z \\
\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y &= -j\omega\mu H_x \\
\frac{\partial}{\partial z}E_x - \frac{\partial}{\partial x}E_z &= -j\omega\mu H_y \\
\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x &= -j\omega\mu H_z
\end{aligned}
$$

$$
\begin{aligned}
E_x &= \frac{-j}{\kappa^2}\left(k_z\frac{\partial}{\partial x}E_z + \omega\mu\frac{\partial}{\partial y}H_z\right) \\
E_y &= \frac{-j}{\kappa^2}\left(k_z\frac{\partial}{\partial y}E_z - \omega\mu\frac{\partial}{\partial x}H_z\right) \\
H_x &= \frac{-j}{\kappa^2}\left(k_z\frac{\partial}{\partial x}H_z - \omega\delta\frac{\partial}{\partial y}E_z\right) \\
H_y &= \frac{-j}{\kappa^2}\left(k_z\frac{\partial}{\partial y}H_z + \omega\delta\frac{\partial}{\partial x}E_z\right)
\end{aligned}
$$

Die homogene ebene Welle (HEW)

$$
\begin{aligned}
+\frac{\partial}{\partial z} e_y &= \mu \frac{\partial}{\partial t} h_x \\
-\frac{\partial}{\partial z} e_x &= \mu \frac{\partial}{\partial t} h_y \\
0 &= \mu \frac{\partial}{\partial t} h_z \\
-\frac{\partial}{\partial z} h_y &= \varepsilon \frac{\partial}{\partial t} e_x \\
+\frac{\partial}{\partial z} h_x &= \varepsilon \frac{\partial}{\partial t} e_y \\
0 &= \varepsilon \frac{\partial}{\partial t} e_z
\end{aligned}
$$

$$\frac{\partial^2}{\partial z^2} e_x - \mu\varepsilon \frac{\partial^2}{\partial t^2} e_x = 0$$

$$e_x(z,t) = \underbrace{c_1\, f_1(z - v\, t)}_{=e_x^+(z,t)} + \underbrace{c_2\, f_2(z + v\, t)}_{=e_x^-(z,t)}$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{\omega}{k}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi\Omega \approx 377\Omega$$

$$h_x^+ = -\frac{e_y^+}{\eta}$$

$$\frac{e_x^+}{h_y^+} = -\frac{e_y^+}{h_x^+} = \eta$$

$$\vec{E} \perp \vec{H} \perp \vec{i}_z$$

$$\frac{e_x^-}{h_y^-} = -\frac{e_y^-}{h_x^-} = -\eta$$

$$
\begin{aligned}
w_\mathrm{e}(t) &= \frac{\varepsilon}{2}(e_x^2 + e_y^2) \\
w_\mathrm{m}(t) &= \frac{\mu}{2}(h_x^2 + h_y^2)
\end{aligned}
$$

$$w_{\mathrm{m}}(t) = \frac{\mu}{2}\frac{1}{\eta^2}(e_x^2 + e_y^2) = w_{\mathrm{e}}(t)$$

$$\begin{aligned} p_x^+ &\equiv 0 \\ p_y^+ &\equiv 0 \\ p_z^+ &= e_x^+ h_y^+ - e_y^+ h_x^+ \end{aligned}$$

$$p_z^+ = \frac{1}{\eta}(e_x^{+\,2} + e_y^{+\,2})$$

$$\overline{\vec{P}(t)} = \overline{\vec{E} \times \vec{H}} = \frac{1}{\eta}\left(\overline{e_x^{+\,2} + e_y^{+\,2}}\right)\vec{i}_z = \frac{1}{2\eta}\left(\overline{E_{x0}^2 + E_{y0}^2}\right)\vec{i}_z$$

$$e_x(z,t) = \mathrm{Re}\{E_x(z)\,e^{j\omega t}\} = E_0\cos\left(k(vt - z)\right) = E_0\cos\left(\omega t - kz\right)$$

$$k = \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\varepsilon\mu}}$$

$$\begin{aligned} \vec{E}_1 = \vec{E}_x &= (E_1\,\vec{i}_x + 0\,\vec{i}_y)\,e^{-jkz} \\ \vec{E}_2 = \vec{E}_y &= (0\,\vec{i}_x + E_2\,\vec{i}_y)\,e^{-jkz} \end{aligned}$$

$$\begin{aligned} e_x(z,t) &= E_1\cos\left(\omega t - kz\right) \\ e_y(z,t) &= E_2\cos\left(\omega t - kz + \psi\right) \end{aligned}$$

$$\begin{aligned} e_x(0,t) &= E_1\cos\left(\omega t\right) \\ e_y(0,t) &= E_2\cos\left(\omega t + \psi\right) \end{aligned}$$

$$\begin{aligned} e_x(0,t) &= E\cos\left(\omega t\right), \\ e_y(0,t) &= E\cos\left(\omega t \mp \frac{\pi}{2}\right) = \pm E\sin\left(\omega t\right) \end{aligned}$$

$$\begin{aligned} e_x(z,0) &= E\cos\left(-kz\right) = E\cos\left(kz\right) \\ e_y(z,0) &= E\cos\left(-kz \pm \frac{\pi}{2}\right) = \pm E\sin\left(kz\right) \end{aligned}$$

$$\begin{aligned} \vec{E} &= E_{y0}\,e^{-jkz}\,\vec{i}_y \\ \eta\,\vec{H} &= -E_{y0}\,e^{-jkz}\,\vec{i}_x \end{aligned}$$

$$\begin{aligned} P &= \int \vec{P}\cdot\mathrm{d}\vec{F} = \frac{1}{2}\mathrm{Re}\{\int(\vec{E}\times\vec{H}^*)\cdot\mathrm{d}\vec{F}\} \\ &= \frac{1}{2}\mathrm{Re}\{\int(E_x H_y^* - E_y H_x^*)\mathrm{d}F\} \\ &= \frac{wd}{2}\mathrm{Re}\{-E_{y0}\,e^{-jkz}(-\frac{E_{y0}}{\eta}\,e^{+jkz})\} = \frac{E_{y0}^2}{2\eta}wd \end{aligned}$$

$$P = \frac{|U|^2}{2Z_{\mathrm{PV}}}$$

$$U = \int_{-d}^{0} E_y \mathrm{d}y = E_{y0}\, d$$

$$P = \frac{|E_{y0}|^2 d^2}{2Z_{\mathrm{PV}}}$$

$$\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H}$$
$$\vec{\nabla} \times \vec{H} = \sigma\vec{E} + j\omega\varepsilon\vec{E}$$

$$\delta = \varepsilon - j\frac{\sigma}{\omega} = \varepsilon(1 - js)$$

$$s = \frac{1}{Q} = \tan\theta = \frac{\sigma}{\varepsilon\omega}$$

$$\eta^2 = \frac{\mu}{\delta} = \frac{\mu}{\varepsilon}\frac{1}{1 - js}$$

$$\eta = \mathbb{R} + j\mathbb{X} = \eta_{\mathrm{E}}\frac{1}{\sqrt{1 - js}}$$

$$jk_z = j\omega\sqrt{\mu\delta} = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - js} = \gamma = \alpha + j\beta$$
$$jk_z = jk_{\mathrm{E}}\sqrt{1 - js}$$

$$\mathbb{R} = \eta_{\mathrm{E}}\sqrt{\frac{\sqrt{1+s^2}+1}{2(1+s^2)}} \qquad \mathbb{X} = \eta_{\mathrm{E}}\sqrt{\frac{\sqrt{1+s^2}-1}{2(1+s^2)}}$$

$$\alpha = k_{\mathrm{E}}\sqrt{\frac{\sqrt{1+s^2}-1}{2}} \qquad \beta = k_{\mathrm{E}}\sqrt{\frac{\sqrt{1+s^2}+1}{2}}$$

$$\eta \approx \eta_{\mathrm{E}}(1 + j\frac{s}{2}) \qquad jk_z \approx k_{\mathrm{E}}(\frac{s}{2} + j)$$

$$\eta \approx \eta_{\mathrm{E}}\frac{1+j}{\sqrt{2s}} \qquad jk_z \approx k_{\mathrm{E}}\sqrt{\frac{s}{2}}(1+j)$$

$$d = \frac{1}{\alpha} \approx \frac{1}{k_{\mathrm{E}}}\sqrt{\frac{2}{s}} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Reflexion an glatten Grenzflächen, die Parallelplattenleitung

$$\frac{\sin\Theta_1}{\sin\Theta_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$$

$$\Gamma_{\mathrm{TM}} = \frac{\sqrt{\varepsilon_2}\cos\Theta_1 - \sqrt{\varepsilon_1}\cos\Theta_2}{\sqrt{\varepsilon_2}\cos\Theta_1 + \sqrt{\varepsilon_1}\cos\Theta_2} = \frac{n^2\cos\Theta_1 - \sqrt{n^2 - \sin^2\Theta_1}}{n^2\cos\Theta_1 + \sqrt{n^2 - \sin^2\Theta_1}}$$

$$T_{\mathrm{TM}} = \frac{2\sqrt{\varepsilon_1}\cos\Theta_1}{\sqrt{\varepsilon_2}\cos\Theta_1 + \sqrt{\varepsilon_1}\cos\Theta_2} = \frac{2n\cos\Theta_1}{n^2\cos\Theta_1 + \sqrt{n^2 - \sin^2\Theta_1}}$$

$$\Gamma_{\mathrm{TE}} = \frac{\sqrt{\varepsilon_1}\cos\Theta_1 - \sqrt{\varepsilon_2}\cos\Theta_2}{\sqrt{\varepsilon_1}\cos\Theta_1 + \sqrt{\varepsilon_2}\cos\Theta_2} = \frac{\cos\Theta_1 - \sqrt{n^2 - \sin^2\Theta_1}}{\cos\Theta_1 + \sqrt{n^2 - \sin^2\Theta_1}}$$

$$T_{\mathrm{TE}} = \frac{2\sqrt{\varepsilon_1}\cos\Theta_1}{\sqrt{\varepsilon_1}\cos\Theta_1 + \sqrt{\varepsilon_2}\cos\Theta_2} = \frac{2\cos\Theta_1}{\cos\Theta_1 + \sqrt{n^2 - \sin^2\Theta_1}}$$

$$\Gamma_{\mathrm{TM}} = 0 \quad \Leftarrow \quad \tan\Theta_{\mathrm{B}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} = n$$

$$\frac{2\pi}{\lambda_1} z_{0m}\cos\Theta_m = m\pi \quad \Rightarrow \quad d = -z_{0m} = \frac{\lambda_1\, m}{2\cos\Theta_m} = \frac{m}{2\, f\,\sqrt{\mu_1\varepsilon_1}\cos\Theta_m}$$

$$\lambda_x = \frac{\lambda_1}{\sin\Theta_m}$$

$$\lambda_{\mathrm{H},m} = \frac{\lambda_0}{\sin\Theta_m}$$

$$d = m\frac{\lambda_0}{2}$$

$$f_{\mathrm{G},m} = \frac{m\, c}{2d}$$

$$\lambda_{\mathrm{G},m} = \frac{c}{f_{\mathrm{G},m}} = \frac{2d}{m}$$

$$\lambda_{\mathrm{H},m} = \frac{\lambda_0}{\sqrt{1 - (\frac{m\,\lambda_0}{2d})^2}}$$

$$\lambda_{\mathrm{H},m} = \frac{\lambda_0}{\sqrt{1 - (\frac{\lambda_0}{\lambda_{\mathrm{G},m}})^2}}$$

$$\sin\Theta_m = \frac{\lambda_0}{\lambda_{\mathrm{H},m}} = \sqrt{1 - (\frac{m\lambda_0}{2d})^2}$$

Die Oberflächenwelle

$$s_1 = \frac{\sigma_1}{\omega\varepsilon_1} \gg 1$$

$$s_2 = \frac{\sigma_2}{\omega\varepsilon_2} \ll 1$$

$$k_{\mathrm{E2}} = \omega\sqrt{\varepsilon_2\mu_0}$$

$$k_z \approx k_{\mathrm{E2}}\Big(1 - j\frac{1}{2s_1}\frac{\varepsilon_2}{\varepsilon_1}\Big)$$

$$\alpha \approx k_{\mathrm{E2}}\frac{1}{2s_1}\frac{\varepsilon_2}{\varepsilon_1} = k_{\mathrm{E2}}\Big(\frac{\mathbb{R}_1}{\mathbb{R}_2}\Big)^2 = \frac{\beta}{2}\frac{1}{s_1}\frac{\varepsilon_2}{\varepsilon_1}$$

$$\beta \approx k_{\mathrm{E2}} = \omega\sqrt{\varepsilon_2\mu_0}$$

$$k_{x1} \approx \sqrt{\frac{\omega\mu_0\sigma_1}{2}}(-1 + j)$$

$$k_{x2} \approx \omega\varepsilon_2\sqrt{\frac{\omega\mu_0}{2\sigma_1}}(1 - j) = \omega\varepsilon_2\eta_1$$

$$\frac{k_{x2}}{k_{x1}} \approx -\frac{\omega\varepsilon_2}{\sigma_1}$$

$$E_{x1} = k_{\mathrm{E2}}\,d_1\,\frac{1 - \frac{j\varepsilon_2}{2s_1\varepsilon_1}}{-1 + j}\,A_1\,e^{jk_{x1}x}\,e^{-jk_z z}$$

$$E_{x1} = -E_{z1}\sqrt{\frac{\omega\varepsilon_2}{2\sigma_1}}(1 + j)$$

$$E_{x2} = -\frac{1}{1 - j}E_{z2}\Big(\sqrt{\frac{2\sigma_1}{\omega\varepsilon_2}} - j\sqrt{\frac{\omega\varepsilon_2}{2\sigma_1}}\Big)$$

$$Z_{\mathrm{W}} = \frac{E_x}{H_y}$$

$$Z_{\mathrm{W2}} = \frac{k_z}{\omega\delta_2} = \frac{k_{\mathrm{E2}}\Big(1 - j\frac{1}{2s_1}\frac{\varepsilon_2}{\varepsilon_1}\Big)}{\omega\varepsilon_2(1 - js_2)} \approx \eta_{\mathrm{E2}}\Big(1 + j\frac{s_2}{2}\Big)$$

$$Z_{\mathrm{W1}} = \frac{k_z}{\omega\delta_1}$$

$$
\begin{aligned}
I_z &= \int_\Sigma \vec{S}_1 \cdot \mathrm{d}\vec{F} = \sigma_1 \int_{x=0}^{\infty} \int_{y=0}^{b} E_{z1} \mathrm{d}x \mathrm{d}y \\
&= \sigma_1 A_1 b\, e^{-jk_z z} \int_{x=0}^{\infty} e^{jk_{x1}\, x} \mathrm{d}x \\
&= j \frac{\sigma_1 A_1 b}{k_{x1}}\, e^{-jk_z z}
\end{aligned}
$$

$$
\mathrm{d}U_z = I_z \mathrm{d}Z
$$

$$
\mathrm{d}P = \frac{1}{2} |I_z|^2 \mathrm{d}Z
$$

$$
\vec{T} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2} \begin{bmatrix} -E_z H_y^* \\ 0 \\ E_x H_y^* \end{bmatrix}
$$

$$
T_{x0} = \frac{1}{2} \Big(- E_{z0} H_{y0}^* \Big) = -\frac{1}{2} \frac{\omega \delta_1^*}{k_{x1}^*} |A_1|^2
$$

$$
\mathrm{d}P = T_{x0}\, b\, \mathrm{d}z
$$

$$
T_{x0}\, b\, \mathrm{d}z = \frac{1}{2} |I_z|^2 \mathrm{d}Z \quad \Rightarrow \quad \mathrm{d}Z = -\frac{\omega \delta_1^* k_{x1}^2}{\sigma_1^2} \frac{\mathrm{d}z}{b}
$$

$$
\mathrm{d}Z \approx \eta_1 \frac{\mathrm{d}z}{b}
$$

$$
\mathrm{d}Z = \mathrm{d}\mathbb{R} + j \mathrm{d}\mathbb{X}, \quad \mathrm{d}\mathbb{R} = \frac{\mathrm{d}z}{b} \mathbb{R}_1, \quad \mathrm{d}\mathbb{X} = \frac{\mathrm{d}z}{b} \mathbb{X}_1
$$

$$
\mathrm{d}P_\mathrm{W} = T_\mathrm{W}\, b\, \mathrm{d}z = \frac{1}{2} |I_z|^2 \mathrm{d}\mathbb{R}
$$

$$
I_z = -b\, H_{y1}(0)
$$

$$
\mathrm{d}P_\mathrm{W} = \frac{1}{2} |H_{y1}(0)|^2 b\, \mathbb{R}_1 \mathrm{d}z \quad \text{bzw.} \quad \frac{\mathrm{d}P_\mathrm{W}}{\mathrm{d}z} = \frac{1}{2} |H_{y1}(0)|^2 b\, \mathbb{R}_1
$$

$$
p = \frac{1}{b} \frac{\mathrm{d}P_\mathrm{W}}{\mathrm{d}z} = \frac{1}{2} |H_\mathrm{tang}(0)|^2 \mathbb{R}_1
$$

$$
\mathbb{R}_1 = \frac{1}{\sigma_1\, d_1} = \mathbb{R}_\square \quad \text{(lies: R square)}
$$

$$
R = \frac{l}{\sigma A}
$$

$$
R = \int \mathrm{d}R = \int_0^l \frac{\mathbb{R}_\square}{b} \mathrm{d}z = \frac{1}{\sigma_1 d_1} \frac{l}{b} \propto \sqrt{\omega}
$$

$$
R = \frac{l}{2\pi\, a} \sqrt{\frac{\omega \mu}{2\sigma_1}}, \quad X = \frac{l}{2\pi\, a} \sqrt{\frac{\omega \mu}{2\sigma_1}}
$$

$$
\frac{R}{R_0} = \frac{X}{X_0} = \frac{a}{2} \sqrt{\frac{\omega \mu \sigma_1}{2}} = \frac{a}{2 d_1} \propto \sqrt{\omega} \gg 1
$$

Resonatoren

$$\lambda_{\mathrm{G},m,n} = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$$

$$\lambda_{\mathrm{H}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_{\mathrm{G}}}\right)^2}}$$

$$v_{\mathrm{P}} = \frac{c}{\sqrt{1 - \left(\frac{\lambda}{\lambda_{\mathrm{G}}}\right)^2}}$$

$$v_{\mathrm{G}} = c\sqrt{1 - (\frac{\lambda}{\lambda_{\mathrm{G}}})^2}$$

$$\kappa^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \equiv \omega^2\varepsilon\mu - k_z^2$$

$$\begin{aligned} P &= \int \mathrm{Re}\{\vec{T}\} \cdot \mathrm{d}\vec{F} = \int T_z \mathrm{d}x\mathrm{d}y = -\frac{1}{2}\int_0^a\int_0^b E_y H_x^* \mathrm{d}x\mathrm{d}y = \frac{\omega k_z \mu}{2}\left(\frac{aA}{\pi}\right)^2 b \int_0^a \sin^2(\frac{\pi}{a}x)\mathrm{d}x \\ &= \frac{\omega k_z \mu}{4} ab \left(\frac{aA}{\pi}\right)^2 \end{aligned}$$

$$-\mathrm{d}P = \frac{1}{2}|H_{\mathrm{tang}}|^2 \mathbb{R}_{\mathrm{M}} \mathrm{d}F$$

$$\begin{aligned} -\frac{\partial}{\partial z}P(z) &= \frac{1}{2}\mathbb{R}_{\mathrm{M}}\Big(2\int_0^a \Big[|H_x|^2 + |H_z|^2\Big]_{y=0}\mathrm{d}x + 2\int_0^b \Big[|\underbrace{H_y}_{0}|^2 + |H_z|^2\Big]_{x=0}\mathrm{d}y\Big) \\ &= \mathbb{R}_{\mathrm{M}} A^2 \Big((\frac{k_z a}{\pi})^2 \int_0^a \sin^2(\frac{\pi}{a}x)\mathrm{d}x + \int_0^a \cos^2(\frac{\pi}{a}x)\mathrm{d}x + \int_0^b \mathrm{d}y \Big) \\ &= A^2 \mathbb{R}_{\mathrm{M}} \Big(\frac{a}{2}\big(1 + (\frac{2a}{\lambda_{\mathrm{H}}})^2\big) + b\Big) \end{aligned}$$

$$\alpha = \frac{\pi}{\omega\mu}\mathbb{R}_{\mathrm{M}}\frac{\lambda_{\mathrm{H}}}{a^3 b}\Big(\frac{a}{2}\big(1 + (\frac{2a}{\lambda_{\mathrm{H}}})^2\big) + b\Big)$$

$$c = p\frac{\lambda_{\mathrm{H}}}{2}, \quad \text{bzw.} \quad k_z = \frac{2\pi}{\lambda_{\mathrm{H}}} = \frac{p\pi}{c}$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2 = \omega_{mnp}^2 \varepsilon\mu$$

$$\omega_{mnp} = \pi v \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$Q_{0,mnp} = \frac{\omega_{mnp} W}{P}$$

$$Q_0 = \frac{2\pi W}{PT} \quad \text{mit} \quad T = \frac{1}{f_{mnp}}$$

$$W = \frac{1}{4} \int_\tau \left(\varepsilon \vec{E} \cdot \vec{E}^* + \mu \vec{H} \cdot \vec{H}^*\right) \mathrm{d}\tau$$

$$P = \frac{1}{2} \mathbb{R}_\mathrm{M} \oint_\Sigma \vec{H}_\mathrm{tang} \cdot \vec{H}^*_\mathrm{tang} \mathrm{d}F$$

$$\lambda_{101} = \frac{2ac}{\sqrt{a^2 + c^2}}$$

$$\omega_{101} = \frac{\pi}{\sqrt{\varepsilon\mu}} \frac{\sqrt{a^2 + c^2}}{ac}$$

$$E_y = -2\omega m \frac{a}{\pi} A \sin\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{c} z\right)$$

$$H_x = 2jk_z \frac{a}{\pi} A \sin\left(\frac{\pi}{a} x\right) \cos\left(\frac{\pi}{c} z\right)$$

$$H_z = -2j A \cos\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{c} z\right)$$

$$W_e = W_m = A^2 \mu \frac{a^2 + c^2}{4c^2} abc$$

$$P = A^2 \mathbb{R}_\mathrm{M} \frac{ac(a^2 + c^2) + 2b(a^3 + c^3)}{c^2}$$

$$Q_0 = \frac{\pi\eta}{2\mathbb{R}_\mathrm{M}} \frac{b\sqrt{(a^2 + c^2)^3}}{ac(a^2 + c^2) + 2b(a^3 + c^3)}$$

$$Q_0 = \frac{\pi\eta\sqrt{2}}{6\mathbb{R}_\mathrm{M}}$$

Koaxialleitungen

$$\vec{E} = E_r \vec{e_r}$$
$$\vec{H} = H_\varphi \vec{e_\varphi}$$

$$\frac{\partial}{\partial z} U(z) + Z' I(z) = 0 \ , \qquad \frac{\partial}{\partial z} I(z) + Y' U(z) = 0$$

$$\text{mit} \qquad Z' = R' + j\omega L' \ , \qquad Y' = G' + j\omega C'$$

$$\frac{\partial}{\partial z} U(z) - Y' Z' U(z) = 0 \ , \qquad \frac{\partial}{\partial z} I(z) - Y' Z' I(z) = 0$$

$$U(z) = U_v e^{-jk_z z} + U_r e^{+jk_z z} \ , \qquad I(z) = I_v e^{-jk_z z} + I_r e^{+jk_z z}$$

$$U_v = Z_\mathrm{L} I_v \ , \qquad U_r = -Z_\mathrm{L} I_r$$

$$Z_\mathrm{L} = \sqrt{\frac{Z'}{Y'}} \qquad \text{bzw.} \qquad Z_\mathrm{L} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$jk_z = \sqrt{Y'Z'} = \sqrt{(G' + j\omega C')(R' + j\omega L')}$$

$$\frac{R'}{L'} = \frac{G'}{C'}$$

$$v_\mathrm{P} = \frac{\omega}{k} = \frac{\omega}{\mathrm{Re}\{k_z\}} \approx \frac{1}{\sqrt{L'C'}}$$

$$L = \frac{1}{I} \int_\mathcal{A} \vec{B} \cdot \vec{n_\mathcal{A}} \ \mathrm{d}\mathcal{A}$$

$$L' = \frac{1}{I} \int_{r_i}^{r_a} \vec{B} \cdot \vec{n_\mathcal{A}} \ \mathrm{d}r = \frac{1}{I} \int_{r_i}^{r_a} B_\varphi \ \mathrm{d}r = \frac{\mu}{I} \int_{r_i}^{r_a} H_\varphi \ \mathrm{d}r$$

$$H_\varphi = \frac{I}{2\pi r}$$

$$L' = \frac{\mu}{I} \int_{r_i}^{r_a} \frac{I}{2\pi r} \ \mathrm{d}r$$

$$L' = \frac{\mu}{2\pi} \ln \frac{r_a}{r_i}$$

$$C = \frac{Q}{\int_\mathcal{C} \vec{E} \cdot \vec{e_r} \ \mathrm{d}r}$$

$$\vec{E} = \frac{\tau}{2\pi\varepsilon} \frac{\vec{e_r}}{r}$$

$$C' = \frac{\tau}{\int_\mathcal{C} \vec{E} \cdot \vec{e_r} \ \mathrm{d}r} = \frac{\tau}{\int_{r_i}^{r_a} \frac{\tau}{2\pi\varepsilon} \frac{\vec{e_r}}{r} \cdot \vec{e_r} \ \mathrm{d}r} = \frac{2\pi\varepsilon}{\int_{r_i}^{r_a} \frac{1}{r} \ \mathrm{d}r}$$

$$C' = \frac{2\pi\varepsilon}{\ln\frac{r_a}{r_i}}$$

$$Z_{\mathrm{L,verlustlos}} = \frac{\eta}{2\pi}\ln\frac{r_a}{r_i} \qquad \text{mit} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\mathbb{R}_\Box = \sqrt{\frac{\omega\mu_{\mathrm{L}}}{2\sigma}}$$

$$R' = \frac{R_{\mathrm{innen}} + R_{\mathrm{aussen}}}{l} \approx \frac{\frac{\mathbb{R}_\Box l}{2\pi r_i} + \frac{\mathbb{R}_\Box l}{2\pi r_a}}{l} = \frac{\mathbb{R}_\Box}{2\pi}(\frac{1}{r_i} + \frac{1}{r_a})$$

$$R' = \sqrt{\frac{\omega\mu_{\mathrm{L}}}{2\sigma}}\frac{1}{2\pi}(\frac{1}{r_i} + \frac{1}{r_a})$$

$$G' = \omega C' \ \tan\delta_\varepsilon = \omega \ \frac{2\pi\varepsilon}{\ln\frac{r_a}{r_i}} \ \tan\delta_\varepsilon$$

$$jk_z = \gamma = \alpha + j\beta = \sqrt{(G' + j\omega C')(R' + j\omega L')}$$

$$\alpha = \alpha_R + \alpha_G = (\underbrace{\frac{R'}{2\sqrt{\frac{L'}{C'}}}}_{(1)} + \underbrace{\frac{G'\sqrt{\frac{L'}{C'}}}{2}}_{(2)}) \underbrace{\frac{1}{\cosh\frac{\delta_R - \delta_G}{2}}}_{(3)}$$

$$\text{mit} \qquad \sinh\delta_R = \frac{R'}{\omega L'} \ , \quad \sinh\delta_G = \frac{G'}{\omega C'}$$

$$\alpha_R \approx \frac{R'}{2\sqrt{\frac{L'}{C'}}} = \frac{\mathbb{R}_\Box}{2\eta r_a} \ \frac{1 + \frac{r_a}{r_i}}{\ln\frac{r_a}{r_i}}$$

$$Z_{\mathrm{L,min.\ Dämpfung}} = \frac{\eta_0}{2\pi\sqrt{\varepsilon_r}}\ln\frac{r_a}{r_i} = \frac{77\ \Omega}{\sqrt{\varepsilon_r}}$$

$$U_{\max} = E_{\max}\ r_i\ \ln\frac{r_a}{r_i} = E_{\max}\ r_a\ \frac{\ln\frac{r_a}{r_i}}{\frac{r_a}{r_i}}$$

$$Z_{\mathrm{L,max.Spannungsfest}} = \frac{60\ \Omega}{\sqrt{\varepsilon_r}}$$

$$P_{\max} = \frac{U_{\max}^2}{2Z_{\mathrm{L}}} = \frac{\pi E_{\max}^2 r_i^2}{\eta}\ln\frac{r_a}{r_i} = \frac{\pi E_{\max}^2 r_a^2}{\eta}\ \frac{\ln\frac{r_a}{r_i}}{(\frac{r_a}{r_i})^2}$$

$$Z_{\mathrm{L,max.\ Leistung}} = \frac{30\ \Omega}{\sqrt{\varepsilon_r}}$$

$$p_v(z) = -\frac{\mathrm{d}P}{\mathrm{d}z} = -\frac{\mathrm{d}}{\mathrm{d}z}P_0 e^{-2\alpha z} = 2\alpha P_0 e^{-2\alpha z} = 2\alpha P(z)$$

$$\text{mit} \qquad \alpha = \alpha_R + \alpha_G$$

Dielektrische Wellenleiter

$$\xi = k_{x1} d$$
$$\eta = k_{x2} d$$

$$-\xi \cot \xi = \eta$$

$$\xi^2 + \eta^2 = \omega^2 \mu_0 d^2 (\varepsilon_1 - \varepsilon_2) = V^2 \quad \Rightarrow \quad V = \frac{2\pi d}{\lambda_0} \sqrt{n_1^2 - n_2^2}$$

$$k_{x1,m} = \frac{(2m-1)\pi}{2d}, \quad m = 1, 2, \ldots$$

$$\omega_{c,m} = \frac{(2m-1)\pi}{2d\sqrt{\varepsilon_0 \mu_0 (\varepsilon_{r1} - \varepsilon_{r2})}}$$

Streifenleitungen

$$Z_\mathrm{L} \approx \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{2\pi} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right)$$

$$Z_\mathrm{W} = \frac{Z_\mathrm{L}}{\sqrt{\varepsilon_\mathrm{eff}}}$$

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_\mathrm{eff}}}$$

$$\varepsilon_\mathrm{eff} = 1 + q(\varepsilon_r - 1)$$

$$f_{c,\mathrm{TEM}} = \frac{c}{4h\sqrt{\varepsilon_r - 1}}$$

$$h_\mathrm{max} = \frac{\lambda_0}{4\sqrt{\varepsilon_r - 1}}$$

$$f_{c,\mathrm{QTEM}} = \frac{c}{(2w + 0,8h)\sqrt{\varepsilon_r}}$$

$$\alpha = \alpha_\mathrm{L} + \alpha_\mathrm{D}$$

$$\begin{aligned} -\frac{\partial}{\partial z} P(z) &= |H_{x0}|^2 \mathbb{R}\, w, \\ \mathbb{R} &= \sqrt{\frac{\omega\mu}{2\sigma}}, \\ \alpha_\mathrm{L} &= \frac{1}{2P(z)} \sqrt{\frac{\omega\mu}{2\sigma}}\, w\, |H_{x0}|^2 = \sqrt{\frac{\omega\mu}{2\sigma}} \frac{1}{\eta h} \end{aligned}$$

$$P = \int \vec{P} \cdot \mathrm{d}\vec{F} = \int T_{w,z} \mathrm{d}x\mathrm{d}y = \frac{1}{2} \int E_y H_x^* \mathrm{d}x\mathrm{d}y = \frac{1}{2\eta} |E_{y0}|^2 h\, w$$

$$Z_\mathrm{W} = \eta \frac{h}{w}.$$

$$\alpha_\mathrm{L} = \sqrt{\frac{\omega\mu}{2\sigma}} \frac{1}{Z_\mathrm{W} w}.$$

$$\alpha_\mathrm{L}' = \alpha_\mathrm{L}\left(1 + \frac{2}{\pi} \arctan(1,4\frac{\Delta}{d_1})\right)$$

$$\alpha_\mathrm{D} = k_\mathrm{E} \frac{s}{2}, \quad \frac{\varepsilon'}{\varepsilon''} = \tan\Theta = s$$

$$\alpha_\mathrm{D} = \frac{\pi}{\lambda} \tan\Theta$$

$$\alpha_\mathrm{D} = \frac{\pi}{\lambda} \tan\Theta \left(\frac{\varepsilon_r}{\varepsilon_\mathrm{eff}} \frac{\varepsilon_\mathrm{eff} - 1}{\varepsilon_r - 1}\right)$$

Wellen und Hindernisse

$$\Gamma_{\mathrm{rauh}} = \Gamma_{\mathrm{glatt}} \exp\left[-2\left(k\sigma\cos\Theta_{\mathrm{e}}\right)^2\right]$$

$$E/E_0 = 1/2 - \exp\left(-j\pi/4\right)\left[C\left(v\right) + jS\left(v\right)\right]/\sqrt{2}$$

$$C\left(v\right) = \int\limits_0^v \cos\left(\pi t^2/2\right) dt \qquad S\left(v\right) = \int\limits_0^v \sin\left(\pi t^2/2\right) dt$$

$$v = h\sqrt{2/\lambda\left(1/d_s + 1/d_e\right)}$$

Antennen

$$\vec{\mathbf{A}}(\vec{r}) = \mu \int\limits_{V'} \frac{\vec{\mathbf{S}}_e(\vec{r}\,')\, e^{-jk|\vec{r}-\vec{r}\,'|}}{4\pi |\vec{r}-\vec{r}\,'|}\, dV'$$

$$\vec{\mathbf{A}}(\vec{r}) = \mu \frac{e^{-jkr}}{4\pi r} \int\limits_{V'} \vec{\mathbf{S}}_e(\vec{r}\,')\; e^{+jkr'\cos\vartheta} dV' = \mu\, \frac{e^{-jkr}}{4\pi r} \vec{\mathbf{N}}\;(\vartheta)$$

$$\begin{aligned}
|\vec{r}-\vec{r}\,'| &= \sqrt{r^2 + r'^2 - 2rr'\;\cos\vartheta} \\
&= \sqrt{(r - r'\cos\vartheta)^2 + r'^2 \sin^2\vartheta} \\
&= (r - r'\cos\vartheta)\left[1 + \frac{1}{2}\frac{r'^2\sin^2\vartheta}{(r - r'\cos\vartheta)^2} + \ldots\right]
\end{aligned}$$

$$\Delta\alpha = k\Delta r = \frac{2\pi}{\lambda}\,\frac{r'^2\sin^2\vartheta}{2\,(r - r'\cos\vartheta)}$$

$$\Delta\alpha_{max} = \frac{\pi}{\lambda}\,\frac{r'^2}{r}$$

$$\frac{\pi}{2} = \frac{\pi}{\lambda}\,\frac{D^2}{r_R}$$

$$r_R = \frac{2D^2}{\lambda}\,(+\lambda)$$

$$\frac{\mathbf{E}_\vartheta(\vartheta,\varphi)}{\mathbf{E}_\vartheta(\vartheta_{max},\varphi_{max})} = \frac{\mathbf{H}_\varphi(\vartheta,\varphi)}{\mathbf{H}_\varphi(\vartheta_{max},\varphi_{max})} = f(\vartheta,\varphi)$$

$$\vartheta_{max} = \pi/2 \;\text{ und}$$

$$\varphi_{max} = \text{beliebig}$$

$$\frac{\mathbf{E}_\vartheta}{\mathbf{E}_\vartheta(\pi/2)} = \frac{\mathbf{H}_\varphi}{\mathbf{H}_\varphi(\pi/2)} = f(\vartheta,\varphi) = \sin\vartheta$$

$$\phi = r^2 Re\left\{\vec{\mathbf{T}}\right\}\cdot\vec{e_r}$$

$$\vec{\mathbf{T}} = \frac{1}{2}\,\vec{\mathbf{E}}\times\vec{\mathbf{H}}^\star$$

$$\frac{1}{2}\int\limits_f \left(\vec{\mathbf{E}}\times\vec{\mathbf{H}}^\star\right)\cdot\vec{e_r}\;df$$

$$P_r = \frac{1}{2} Re \left\{ \iint_f \left(\vec{\mathbf{E}} \ \times \vec{\mathbf{H}}^\star \right) \cdot \vec{e_r} \ df \right\}$$

$$df = r^2 \sin \vartheta d\vartheta d\varphi = r^2 d\Omega$$

$$P_r = \int_{4\pi} \ Re \left\{ \vec{\mathbf{T}} \right\} r^2 \cdot \vec{e}_r \ d\Omega = \int_{4\pi} \ \phi \ d\Omega = \phi_{max} \ \int_{4\pi} \frac{\phi}{\phi_{max}} \ d\Omega$$

$$f\left(\vartheta, \varphi\right) = \frac{\mathbf{E}\left(\vartheta, \varphi\right)}{\mathbf{E}\left(\vartheta_{max}, \varphi_{max}\right)}$$

$$\frac{\phi}{\phi_{max}} = \left| f\left(\vartheta, \varphi\right) \right|^2$$

$$P_r = \phi_{max} \ \int_{4\pi} \ \left| f\left(\vartheta, \varphi\right) \right|^2 d\Omega = \phi_{max} \Omega_{\ddot{a}}$$

$$\Omega_{\ddot{a}} = \int_0^{2\pi} \ \int_0^{\pi} \ \left| f\left(\vartheta, \varphi\right) \right|^2 \sin \vartheta d\vartheta d\varphi$$

$$D = \frac{4\pi}{\Omega_{\ddot{a}}} = \frac{4\pi}{\int\limits_0^{2\pi} d\varphi \int_0^{\pi} |f(\vartheta, \varphi)|^2 \sin \vartheta d\vartheta}$$

$$\frac{P_{\mathrm{L_{HD}}}}{P_{\mathrm{L_{DUT}}}} = \frac{P_{\mathrm{r_{HD}}}}{P_{\mathrm{r_{DUT}}}} e_{\mathrm{DUT}} = \cdots$$

$$\left| \mathbf{E}_{\vartheta_{HD}} \right| = \frac{\eta \left| \mathbf{I} \right| s}{2\lambda r} \ \sin \vartheta.$$

$$P_{\mathrm{r_{HD}}} = \frac{\pi \eta}{3} \ \left(\frac{s^2}{\lambda^2} \right) \ \left| \mathbf{I} \right|^2$$

$$\left| \mathbf{E}_{\vartheta_{HD}} \right| = \sqrt{\frac{3 \ \eta}{4 \ \pi}} \ \sqrt{P_{r,HD}} \ \frac{\sin \vartheta}{r}$$

$$P_{\mathrm{r_{HD}}} = \frac{4\pi r^2}{3\eta} \ \left| \mathbf{E}_{\vartheta_{HD}} \right|^2 \ \frac{1}{\sin^2 \vartheta}$$

$$\frac{\left| \mathbf{E}_{\vartheta_{HD}} \right|}{\left| \mathbf{E}_{\vartheta_{\mathrm{HD}}} \right|_{max}} = f_{HD}\left(\vartheta\right) = \sin \ \vartheta$$

$$P_{\mathrm{r_{HD}}} = \frac{4\pi r^2}{3\eta} \ \left| \mathbf{E}_{\vartheta, HD} \right|^2_{max}$$

$$\mathbf{E}_\vartheta = \frac{j\eta \mathbf{I}}{2\pi r} \ e^{-jkr} \ \mathbf{F}\left(\vartheta, \varphi\right)$$

$$\mathbf{H}^\star_\varphi = -j \ \frac{\mathbf{I}^\star}{2\pi r} \ e^{+jkr} \mathbf{F}^\star \left(\vartheta, \varphi\right)$$

$$P_{\mathrm{r_{DUT}}} = \frac{1}{2} Re \left\{ \oint_f \ \left(\vec{\mathbf{E}} \times \vec{\mathbf{H}}^\star \right) \cdot \vec{e_r} \ df \right\} = \frac{1}{2} \ \int_0^{2\pi} \ \int_0^{\pi} \ \eta \ \frac{\left| \mathbf{I} \right|^2}{4\pi^2 r^2} \left| \mathbf{F}\left(\vartheta, \varphi\right) \right|^2 r^2 \sin \vartheta d\vartheta d\varphi =$$

$$= \eta \frac{|\mathbf{I}|^2}{8\pi^2} \int_0^{2\pi} \int_0^{\pi} |\mathbf{F}(\vartheta, \varphi)|^2 \sin\vartheta d\vartheta d\varphi$$

$$|\mathbf{E}_\vartheta|_{max} = \frac{\eta |\mathbf{I}|}{2\pi r} |\mathbf{F}(\vartheta_{max}, \varphi_{max})|$$

$$\frac{|\mathbf{E}_\vartheta|}{|\mathbf{E}_\vartheta|_{max}} = \frac{|\mathbf{F}(\vartheta, \varphi)|}{|\mathbf{F}(\vartheta_{max}, \varphi_{max})|} = |f(\vartheta, \varphi)|$$

$$|\mathbf{F}(\vartheta, \varphi)| = |f(\vartheta, \varphi)| \, |\mathbf{F}(\vartheta_{max}, \varphi_{max})| = |f(\vartheta, \varphi)| \, \frac{2\pi r}{\eta |\mathbf{I}|} \, |\mathbf{E}_\vartheta|_{max}$$

$$P_{\mathrm{r_{DUT}}} = \frac{r^2}{2\eta} \, |\mathbf{E}_\vartheta|^2_{max} \int_0^{2\pi} \int_0^{\pi} |f(\vartheta, \varphi)|^2 \sin\vartheta d\vartheta d\varphi$$

$$\frac{P_{\mathrm{r_{HD}}}}{P_{\mathrm{rDUT}}} = \frac{\frac{4\pi r^2}{3\eta}}{\frac{r^2}{2\eta}} \frac{|\mathbf{E}_{\vartheta_{HD}}|^2_{max}}{|\mathbf{E}_\vartheta|^2_{max} \int_0^{2\pi} \int_0^{\pi} |f(\vartheta, \varphi)|^2 \sin\vartheta d\vartheta d\varphi}$$

$$G_{REF} = \frac{P_{\mathrm{L_{REF}}}}{P_{\mathrm{L_{DUT}}}} \cdot \frac{|\mathbf{E}_{\vartheta_{DUT}}|^2_{max}}{|\mathbf{E}_{\vartheta_{REF}}|^2_{max}}$$

$$G_{HD} = e_{DUT} \, \frac{8\pi/3}{\int_0^{2\pi} \int_0^{\pi} |f(\vartheta, \varphi)|^2 \sin\vartheta d\vartheta d\varphi}$$

$$G_{\mathrm{HD}} = \frac{P_{\mathrm{L_{HD}}}}{P_{\mathrm{L_{DUT}}}} \cdot \frac{|E_{\vartheta_{\mathrm{DUT}}}|^2_{\mathrm{max}}}{|E_{\vartheta_{\mathrm{HD}}}|^2_{\mathrm{max}}} = e_{\mathrm{DUT}} \, \frac{P_{\mathrm{r_{HD}}}}{P_{\mathrm{r_{DUT}}}} \cdot \frac{|E_{\vartheta_{\mathrm{DUT}}}|^2_{\mathrm{max}}}{|E_{\vartheta_{\mathrm{HD}}}|^2_{\mathrm{max}}}$$

$$G_{HD} = \frac{8\pi/3}{\int_0^{2\pi} \int_0^{\pi} |f(\vartheta, \varphi)|^2 \, \sin\vartheta d\vartheta d\varphi}$$

$$G_{HD} = \frac{8\pi/3}{\Omega_{ä}} = \frac{2}{3} \, G_{ISO}$$

$$T_E(r) = G \, \frac{P_S}{4\pi r^2}$$

$$|\mathbf{E}_E| = \frac{A}{\lambda r} \, E_0$$

$$T_E(r) = \frac{|\mathbf{E}_E|^2}{2\eta} = \left(\frac{A}{\lambda r}\right)^2 \, \frac{E_0^2}{2\eta}$$

$$P_S = \frac{E_0^2}{2\eta} \, A$$

$$G = 4\pi r^2 \, \frac{T_E(r)}{P_S} = 4\pi \, \frac{A}{\lambda^2}$$

$$G_{ISO} = \frac{4\pi}{\lambda^2} \, A \, w$$

$$P_E = A \, T_E$$

$$G_{\text{DUT/ISO}} = \frac{EIRP}{P_{\text{L}_{\text{DUT}}}} \cdot \frac{|E_{\vartheta_{\text{DUT}}}|^2_{\text{max}}}{|E_{\vartheta_{\text{ISO}}}|^2_{\text{max}}}$$

$$EIRP = P_L G_{ISO}$$

$$L = ns$$

$$l = \frac{\pi D}{\cos\psi}$$

$$s = l\sin\psi = \pi D\tan\psi$$

$$k_{\text{wendel}} l - k_0 s = 2\pi\nu \qquad \nu = 1, 2, 3, \ldots$$

$$k_{\text{wendel}} = \frac{\omega}{v}$$

$$\omega\left(\frac{l}{v} - \frac{s}{c_0}\right) = 2\pi\nu \qquad \nu = 1, 2, 3, \ldots$$

$$\omega = 2\pi\frac{c_0}{\lambda_0} \quad und \quad l \approx \pi D \approx \lambda_0$$

$$l = (\lambda_0 + s)\,\frac{v}{c_0}$$

$$\frac{3}{4}\,\lambda_0 < \lambda < \frac{4}{3}\,\lambda_0$$

$$\mathbf{P} = \frac{1}{2}\;|\mathbf{I}|^2\,\mathbf{Z}_A$$

$$\vec{\mathbf{T}} = \frac{1}{2}\,\vec{\mathbf{E}} \times \vec{\mathbf{H}}^\star$$

$$\mathbf{T}_r = \frac{1}{2}\,\mathbf{E}_\vartheta \mathbf{H}_\varphi^\star$$

$$\mathbf{T}_\vartheta = -\frac{1}{2}\,\mathbf{E}_r \mathbf{H}_\varphi{}^\star \approx 0$$

$$\mathbf{T}_\varphi \equiv 0$$

$$|\mathbf{H}_\varphi| = |\mathbf{E}_\vartheta|\,/\eta$$

$$P_r = \frac{1}{2}\int\limits_{\varphi=0}^{2\pi}\int\limits_{\vartheta=0}^{\pi}\frac{|\mathbf{E}_\vartheta|^2}{\eta}\,r^2\sin\vartheta d\vartheta d\varphi$$

$$\mathbf{E}_\vartheta = j\eta\,\frac{\mathbf{I}s}{2\lambda}\,\frac{e^{-jkr}}{r}\;\sin\vartheta$$

$$P_r = \eta\,\frac{|\mathbf{I}|^2\,s^2}{8\lambda^2}\,2\pi\int\limits_0^\pi \sin^3\vartheta d\vartheta$$

$$P_r = \frac{1}{3}\pi\eta\,\left(\frac{s^2}{\lambda^2}\right)\,|\mathbf{I}|^2$$

$$R_A = \frac{2}{3}\,\pi\eta\left(\frac{s^2}{\lambda^2}\right)$$

$$m = \frac{1 + |\rho|}{1 - |\rho|} = \frac{|U_{max}|}{|U_{min}|}$$

$$\mathbf{Z}(z) = \frac{\mathbf{U}(z)}{\mathbf{I}(z)}$$

$$Q = \frac{\omega}{2R_A}\left(\frac{\partial X_A}{\partial \omega}\right)\Big|_{\omega=\omega_0} \qquad \text{mit} \qquad \mathbf{Z}_A = R_A + jX_A$$

$$\Delta\omega = \frac{\omega_0}{Q}$$

$$Q = \frac{\omega}{2G_A}\left(\frac{\partial B_A}{\partial \omega}\right)\Big|_{\omega=\omega_0} \qquad \text{mit} \qquad \mathbf{Y}_A = G_A + jB_A$$

$$r = r_0 \exp\left(a\psi\right)$$

$$\frac{P_{S1}}{P_{E2}} = \frac{P_{S2}}{P_{E1}}$$

$$\begin{aligned} G\left(\vartheta,\varphi\right) &= \frac{P_{\mathrm{L_{REF}}}}{P_{\mathrm{L_{DUT}}}} \cdot \frac{\left|E_{\vartheta_{\mathrm{DUT}}}\left(\vartheta,\varphi\right)\right|^2}{\left|E_{\vartheta_{\mathrm{REF}}}\left(\vartheta,\varphi\right)\right|^2} = \\ &= \frac{P_{\mathrm{L_{REF}}}}{P_{\mathrm{L_{DUT}}}} \frac{\left|E_{\vartheta_{\mathrm{DUT}}}\right|^2_{\max}}{\left|E_{\vartheta_{\mathrm{REF}}}\right|^2_{\max}} \frac{\left|f_{\mathrm{DUT}}\left(\vartheta,\varphi\right)\right|^2}{\left|f_{\mathrm{REF}}\left(\vartheta,\varphi\right)\right|^2} = \\ &= G_{\mathrm{REF}} \cdot \frac{\left|f_{\mathrm{DUT}}\left(\vartheta,\varphi\right)\right|^2}{\left|f_{\mathrm{REF}}\left(\vartheta,\varphi\right)\right|^2} \end{aligned}$$

$$MEG = \int\limits_{4\pi} G\left(\vartheta,\varphi\right)\; P\left(\vartheta,\varphi\right) d\Omega$$

Wellen im freien Raum

$$r \doteq \sqrt{\frac{d\lambda}{4}}$$

$$T_{e,ISO} = \frac{P_s}{4\,\pi\,d^2}$$

$$T_e = \frac{P_s\,G_s}{4\,\pi\,d^2}$$

$$P_e = T_e\,A_e$$

$$P_e = T_e\,A_e = \frac{P_s\,G_s}{4\,\pi\,d^2} A_e$$

$$A = \frac{\lambda^2}{4\,\pi}\,G_{iso}$$

$$P_e = \frac{P_s\,G_s}{4\,\pi\,d^2}\,\frac{\lambda^2}{4\,\pi}\,G_e = P_s\,\left(\frac{\lambda}{4\,\pi\,d}\right)^2\,G_s\,G_e$$

$$P_e = P_s\,\left(\frac{1}{\lambda\,d}\right)^2\,A_s\,A_e$$

$$L\,\Big|_{\mathrm{dB}} = 10\,\log\,\frac{P_s}{P_e}$$

$$P_e\,\Big|_{\mathrm{dBW}} = P_s\,\Big|_{\mathrm{dBW}} + G_s\,\Big|_{\mathrm{dB}} - L_{ISO}\Big|_{\mathrm{dB}} + G_e\,\Big|_{\mathrm{dB}}$$

$$L_{ISO} = -20\,\log\,\left(\frac{\lambda}{4\pi\,d}\right)$$

$$L_s = 10\,\log\,\frac{P_s}{P_n} = 10\,\cdot\,\log\,\frac{P_s}{P_{e,\min}}\,\frac{P_{e,\min}}{P_n} = L\Big|_{\mathrm{dB}} + SNR_{\min}\,\Big|_{\mathrm{dB}}$$

$$T_i = \frac{P_s\,G_s}{4\pi d^2}$$

$$P_e = T_e\,A_e = \frac{T_i\,\sigma}{4\pi d^2}\,A_e = \frac{P_s\,G_s\,\sigma}{\left(4\pi d^2\right)^2}\,\frac{\lambda^2}{4\pi}\,G_e$$

$$\frac{P_e}{P_s} = \sigma\,G_s^2\,\left(\frac{\lambda}{4\pi}\right)^2\,\frac{1}{4\pi d^4}$$

$$\sigma = AG = A\frac{4\pi}{\lambda^2}\,A = 4\,\pi\frac{A^2}{\lambda^2}$$

Mehrwegeausbreitung

$$\tau_1 = d_1/c, \qquad \text{und} \qquad \tau_2 = d_2/c$$

$$\mathbf{h}(\tau) = \mathbf{A}_1\,\delta\left(\tau - \tau_1\right) + \mathbf{A}_2\,\delta\left(\tau - \tau_2\right)$$

$$\mathbf{H}\left(j\omega\right) = \int\limits_0^\infty \mathbf{h}(\tau)\,e^{-j\omega\tau}\,d\tau = \mathbf{A}_1\,e^{-j\omega\tau_1} + \mathbf{A}_2\,e^{-j\omega\tau_2}$$

$$\left|\mathbf{H}\left(j\omega\right)\right| = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\left(\omega\cdot\Delta\tau\right)} \qquad \text{mit} \qquad \Delta\tau = \tau_2 - \tau_1$$

$$\Delta\,f_{Notch} = \frac{1}{\Delta\,\tau}$$

$$\mathbf{H}\left(j\omega\right) = \left|\mathbf{H}\left(j\omega\right)\right|\;e^{j\phi_H\,(j\omega)}$$

$$\tau_{Gr} = -\frac{d\phi_H}{d\omega}$$

$$\vec{\mathbf{E}}\left(\vec{r}\right) = \vec{\mathbf{E}}_1\,e^{-j\vec{k}_1\vec{r}} + \vec{\mathbf{E}}_2\,e^{-j\vec{k}_2\vec{r}}$$

$$\vec{\mathcal{E}}(t) = \vec{E}_0\cdot\cos\left(\omega t - kd\right)$$

$$\begin{aligned}
\vec{\mathcal{E}}\left(t\right) &= \vec{E}_0\cdot\cos\left(\omega t - k\left[d_0 + vt\right]\right)\\
&= \vec{E}_0\cdot\cos\left(t\left[\omega - kv\right] - kd_0\right)\\
&= \vec{E}_0\cdot\cos\left(t\left[2\pi f - \frac{2\pi}{\lambda}v\right] - kd_0\right)\\
&= \vec{E}_0\cdot\cos\left(2\pi t\left[f - \frac{v}{\lambda}\right] - kd_0\right)
\end{aligned}$$

$$\Delta f_D = -\frac{v}{\lambda} = -f\cdot\frac{v}{c}$$

$$\Delta f_D = -\frac{v}{\lambda}\cos\left(\gamma\right) = -f\cdot\frac{v}{c}\,\cos\left(\gamma\right)$$

$$p(E) = \frac{1}{\sigma\sqrt{2\cdot\pi}}\cdot e^{-\frac{E^2}{2\cdot\sigma^2}}$$

$$\text{Varianz:} = \overline{E^2} - \left(\overline{E}\right)^2$$

$$\text{Varianz} = \overline{E^2} = \int\limits_{-\infty}^{\infty} E^2\cdot p(E)dE = \sigma^2$$

$$\sigma^2 = \overline{Re(\mathbf{E})^2} = P_m$$

$$p(a) = \frac{a}{\sigma^2}\cdot\exp\left[-\frac{a^2}{2\sigma^2}\right]$$

$$\begin{array}{rl} \text{Mittelwert} & \overline{a} = \sigma\sqrt{\frac{\pi}{2}} \\ \text{quadrat. Mittelwert} & \overline{a^2} = 2\sigma^2 \\ \text{Varianz} & \overline{a^2} - (\overline{a})^2 = 2\,\sigma^2 - \sigma^2\frac{\pi}{2} = 0.429\sigma^2 \\ \text{Medianwert} & a_{50} = \sigma\sqrt{2\cdot ln2} = 1.18\,\sigma \end{array}$$

$$p(a) = \frac{a}{\sigma^2}\cdot\exp\left[-\frac{a^2+A^2}{2\sigma^2}\right]\cdot I_0\left(\frac{aA}{\sigma^2}\right)$$

$$\text{quadrat. Mittelwert} \quad \overline{a^2} = 2\sigma^2 + A^2$$

$$\frac{P_e}{P_r} = G_s\cdot G_e\left(\frac{\lambda}{4\pi d_0}\right)^2\left(\frac{d_0}{d}\right)^n$$

$$p(F) = \frac{1}{\sigma_F\sqrt{2\cdot\pi}}\cdot\exp\left[-\frac{(F-M)^2}{2\cdot\sigma_F^2}\right]$$