Name: ____

Matriculation number: ______________________________

Study code: ◯ 066 437 (master TC) ◯ 066 938 (master CE)
◯ 033 235 (bachelor EE) ◯ other: ___________

◯ I have read and understood the instructions below.

Instructions (READ CAREFULLY)

- This problem statement has to be handed in together with your solutions.
- A copy of the lecture notes is provided. You may use these lecture notes and a simple calculator during the exam. The use of any other documents or electronic devices is not permitted.
- Begin your solution to each problem on a new sheet of paper. If you do not attempt a problem, turn in a blank page on which you have written the missing problem number.
- Be sure to hand in your solutions by arranging them in such a way that the problems 1-4 appear in order.
- Check your results for plausibility. If you arrive at a result which is obviously incorrect, indicate that you are aware that the result is incorrect and elaborate, if possible.
- If the answer to one part depends upon the results of earlier parts that you were not able to answer, demonstrate your competence on the remaining parts by making reasonable assumptions about answers to the missing parts.
- When you are asked to make a sketch, do not forget to label the axes and mark significant points (e.g., maxima, minima, zeros, etc.).
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary.
- Note that the problems are weighted differently.
- The result of this exam will be sent to your generic student email address (e<matriculation_number>@student.tuwien.ac.at) within two weeks.

Problem	1	2	3	4	Sum
Max. credits	18	17	18	17	70
Credits					

Exercise credits: **Total credits:** **Grade (written part):**

Problem 1 (18 points) Consider the real-valued random process

$$\mathsf{x}[n] = A\sin(\theta_0 n + \Phi),$$

where A and θ_0 are deterministic and $\Phi \sim \mathcal{U}(-\pi, \pi)$ is random. Let $\mathsf{y}[n] = \mathsf{x}^2[n]$.

a) Calculate the mean and autocorrelation function of $\mathsf{x}[n]$.

b) Calculate the mean and autocorrelation function of $\mathsf{y}[n]$.

c) Calculate the cross-correlation function of $\mathsf{x}[n]$ and $\mathsf{y}[n]$.

d) Are $\mathsf{x}[n]$ and $\mathsf{y}[n]$ individually wide-sense stationary (WSS) or individually wide-sense cyclostationary? Justify your answer.

e) If $\mathsf{y}[n]$ is WSS, calculate and sketch the power spectral density (PSD) $S_{\mathsf{y}}(e^{j\theta})$ and calculate the mean power P_{y}.
If $\mathsf{y}[n]$ is wide-sense cyclostationary, calculate and sketch the PSD $S_{\tilde{\mathsf{y}}}(e^{j\theta})$ and calculate the mean power $P_{\tilde{\mathsf{y}}}$ of the stationarized random process $\tilde{\mathsf{y}}[n]$.

f) Are $\mathsf{x}[n]$ and $\mathsf{y}[n]$ jointly WSS? Justify your answer.

Problem 2 (17 points) Let $\mathbf{x} = (\mathsf{x}_1\ \mathsf{x}_2\ \mathsf{x}_3)^T \sim \mathcal{N}(\boldsymbol{\mu}_{\mathbf{x}}, \boldsymbol{I})$ be a Gaussian random vector. The random vector $\mathbf{y} = \boldsymbol{A}\mathbf{x}$ is derived from $\mathbf{x}$ such that the covariance matrix of $\mathbf{y}$ is

$$\boldsymbol{C}_{\mathbf{y}} = \begin{pmatrix} \gamma & \alpha & 0 \\ \alpha & \gamma & \beta \\ 0 & \beta & \gamma \end{pmatrix}, \quad \alpha \neq 0,\ \beta \neq 0.$$

Hint: $\lambda_1 = \gamma + \sqrt{\alpha^2 + \beta^2}$ *and* $\lambda_2 = \gamma$ *are eigenvalues of* $\boldsymbol{C}_{\mathbf{y}}$.

a) Specify the pdf of $\mathbf{x}$ (simplify the expression as much as possible).

b) Are the elements of $\mathbf{x}$ statistically independent? Justify your answer.

c) Specify the range of values allowed for γ. Furthermore, specify the range of values allowed for α and β (in terms of γ).

d) Find the eigenvalue λ_3 of $\boldsymbol{C}_{\mathbf{y}}$.

e) For $\gamma = 2$ and $\alpha = \beta = 1/\sqrt{2}$, find the matrix $\boldsymbol{A}$.

f) Are any elements of $\mathbf{y}$ mutually independent? If yes, which? Justify your answer.

Problem 3 (18 points) Consider a Gaussian random variable $\mathsf{a} \sim \mathcal{N}(0, \sigma^2)$ and a discrete random variable $\mathsf{b} \in \{-1, 1\}$ with $\mathrm{P}\{\mathsf{b}=1\} = 1/2$. The random variables a and b are statistically independent.

a) Calculate the joint pdf $f_{\mathsf{a},\mathsf{v}}(a, v)$ for $\mathsf{v} = \mathsf{ab}$.

b) Calculate the marginal pdf $f_{\mathsf{v}}(v)$.

c) Find out whether a and v are statistically independent and/or uncorrelated and/or orthogonal. Justify your answers.

d) Repeat subtasks a) – c) for the case where $\mathsf{a} \in \{-1, 1\}$ is a discrete random variable with $\mathrm{P}\{\mathsf{a}=1\} = 1/2$ and b is a Gaussian distributed random variable with $\mathcal{N}(0, \sigma^2)$.

Problem 4 (17 points) Let $\mathsf{x} \sim \mathcal{U}(a, b)$ be a random variable which is uniformly distributed in the interval $[a, b]$.

a) Specify and sketch the pdf and cdf of x.

b) Calculate the moments $m_{\mathsf{x}}^{(1)}, m_{\mathsf{x}}^{(2)}$ and the central moments $m_{\mathsf{x}-\mu_{\mathsf{x}}}^{(1)}, m_{\mathsf{x}-\mu_{\mathsf{x}}}^{(2)}$.

c) Let $a = -1.5$, $b = 1.5$ and consider the transformation $\mathsf{y} = g(\mathsf{x})$, where

$$g(x) = \sum_{k=-\infty}^{\infty} h(x - 2k), \qquad \text{with} \qquad h(x) = \begin{cases} x^2, & \text{for } |x| \le 1 \\ 0, & \text{for } |x| > 1 \end{cases}.$$

i) Sketch $g(x)$ for $|x| \le 1.5$.

ii) Calculate and sketch the pdf of y.

iii) Calculate $\mathrm{P}\{\mathsf{y} > 1/4\}$.