Lösungen zur 1. Teilprüfung
Signale und Systeme 2, Sommersemester 2007

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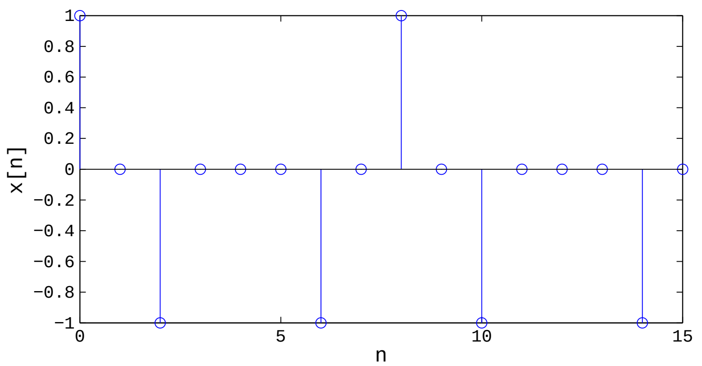
1. Beispiel:

Gruppe A

a) $N_1 = 4,\ N_2 = 8$

b) $x_1[n] = x_1[-n], \ \rightarrow\ x_1[n]$ gerades Signal, detto $x_2[n]$

c) $N = 8$

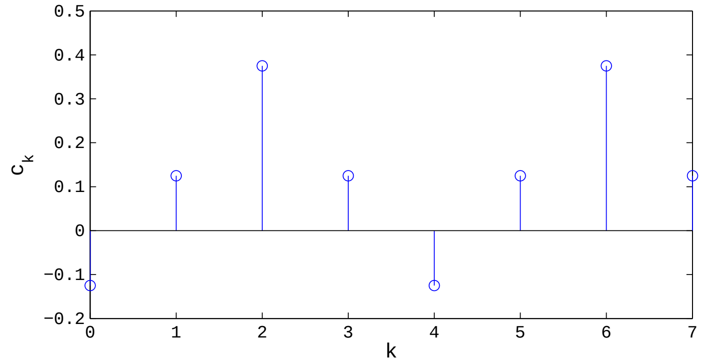


d) $c_k = \frac{1}{8}[-1, 1, 3, 1, -1],\ c_{8-k} = c_k,\ k = 0, 1, 2, 3$
oder mit Einsimpulsen: $c_k = \frac{1}{8}(-\delta[k] + \delta[k-1] + 3\delta[k-2] + \delta[k-3] - \delta[k-4])$

d$_1$) $x[n]$ ist reell und gerade $\longrightarrow$ c_k ist reell

d$_2$) $x[n]$ ist reell und gerade $\longrightarrow$ c_k ist gerade

d$_3$)



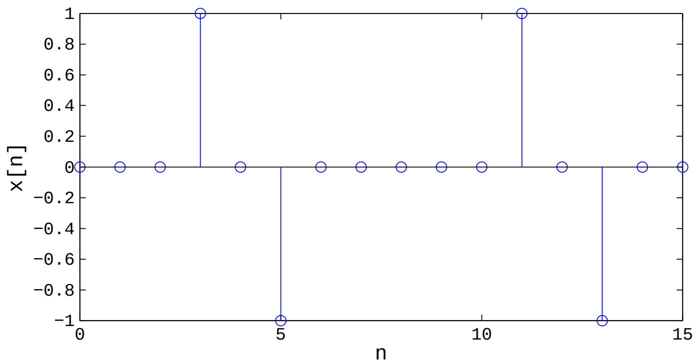
Gruppe B

a) $N_1 = 4,\ N_2 = 8$

b) $x_1[n] = -x_1[-n], \;\rightarrow\; x_1[n]$ ungerades Signal

$x_2[n] = x_2[-n], \;\rightarrow\; x_1[n]$ gerades Signal

c) $N = 8$



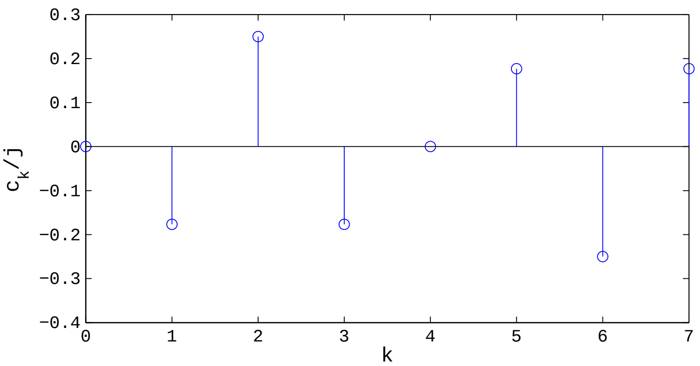
d) $c_k = \frac{j}{4}\left[0, -\frac{1}{\sqrt{2}}, 1, -\frac{1}{\sqrt{2}}, 0\right], \; c_{8-k} = c_k^*, \; k = 0, 1, 2, 3$

oder mit Einsimpulsen: $c_k = \frac{j}{4}\left(-\frac{1}{\sqrt{2}}\delta[k-1] + \delta[k-2] - \frac{1}{\sqrt{2}}\delta[k-3]\right)$

d_1) $x[n]$ ist reell und ungerade $\rightarrow$ c_k ist imaginär, $c_0 = c_4 = 0$ ist reell

d_2) $x[n]$ ist reell und ungerade $\rightarrow$ $\Im m\{c_k\}$ ist ungerade

d_3)



2. Beispiel:

Gruppe A

a) $x[n] = x_1[n] = \cos\frac{\pi}{2}n = \frac{1}{2}e^{j\frac{\pi}{2}n} + \frac{1}{2}e^{-j\frac{\pi}{2}n}$

und damit $X\left(e^{j\theta}\right) = \pi\delta\left(\theta - \frac{\pi}{2}\right) + \pi\delta\left(\theta + \frac{\pi}{2}\right), \; \theta \in [-\pi, \pi]$, Skizze ist trivial

Alternative: $X\left(e^{j\theta}\right)$ mit Fourierreihenkoeffizienten $c_k = \frac{1}{4}\left(1 - (-1)^k\right)$, $k = 0, 1, 2, 3$ von $x[n] = x_1[n]$ berechnen.

b) Skizze von $H\left(e^{j\theta}\right)$ ist trivial

c) $h[n] = \frac{1}{2\pi}\int_0^{\pi} e^{jn\theta}d\theta = \frac{j}{2\pi n}\left(1-(-1)^n\right)$

$\Re e\{h[n]\} = \frac{1}{2}\delta[n]$

$\Im m\{h[n]\} = \begin{cases} 0 & n=0,\, n \text{ gerade} \\ \frac{1}{\pi n} & n \text{ ungerade} \end{cases}$

d) $Y\left(e^{j\theta}\right) = H\left(e^{j\theta}\right)X\left(e^{j\theta}\right) = \pi\delta\left(\theta - \frac{\pi}{2}\right),\ \theta \in [-\pi,\pi]$, Skizze ist trivial

e) $y[n] = \frac{1}{2}e^{j\frac{\pi}{2}n}$

Gruppe B

a) $x[n] = x_1[n] = -\sin\frac{\pi}{2}n = -\frac{1}{2j}e^{j\frac{\pi}{2}n} + \frac{1}{2j}e^{-j\frac{\pi}{2}n}$

und damit $X\left(e^{j\theta}\right) = j\pi\delta\left(\theta-\frac{\pi}{2}\right) - j\pi\delta\left(\theta+\frac{\pi}{2}\right),\ \theta\in[-\pi,\pi]$, Skizze ist trivial

Alternative: $X\left(e^{j\theta}\right)$ mit Fourierreihenkoeffizienten $c_k = \frac{j^k}{4}\left(1-(-1)^k\right)$, $k=0,1,2,3$ von $x[n] = x_1[n]$ berechnen.

b) Skizze von $H\left(e^{j\theta}\right)$ ist trivial

c) $h[n] = \frac{1}{2\pi}\int_{-\pi}^{0} e^{jn\theta}d\theta = -\frac{j}{2\pi n}\left(1-(-1)^n\right)$

$\Re e\{h[n]\} = \frac{1}{2}\delta[n]$

$\Im m\{h[n]\} = \begin{cases} 0 & n=0,\, n \text{ gerade} \\ -\frac{1}{\pi n} & n \text{ ungerade} \end{cases}$

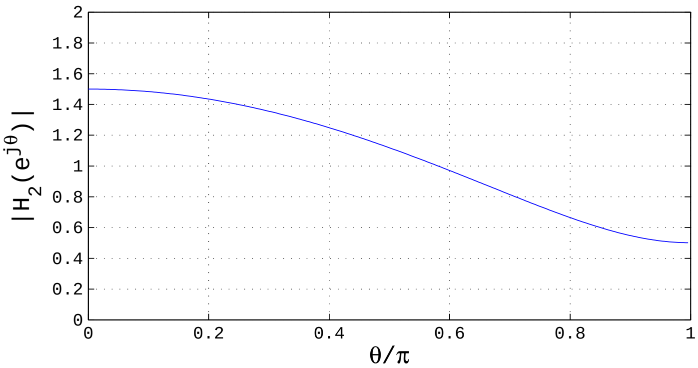
d) $Y\left(e^{j\theta}\right) = H\left(e^{j\theta}\right)X\left(e^{j\theta}\right) = -j\pi\delta\left(\theta + \frac{\pi}{2}\right),\ \theta \in [-\pi,\pi]$, Skizze ist trivial

e) $y[n] = -\frac{j}{2}e^{-j\frac{\pi}{2}n}$

3. Beispiel:

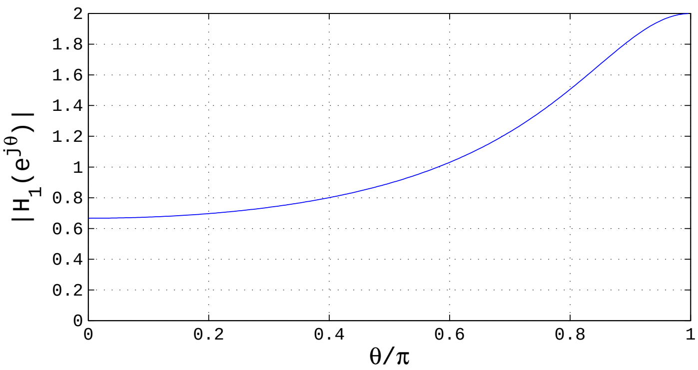
Gruppe A

a) $H_2\left(e^{j\theta}\right) = 1 + \frac{1}{2}e^{-j\theta}, \quad H_2\left(e^{j0}\right) = \frac{3}{2},\ H_2\left(e^{j\pi}\right) = \frac{1}{2}$

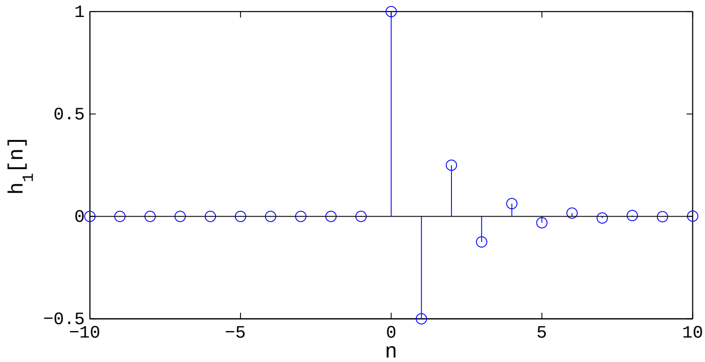


b) $H_1(e^{j\theta})H_2(e^{j\theta}) = 1, \rightarrow \; H_1(e^{j\theta}) = \dfrac{1}{H_2(e^{j\theta})} = \dfrac{1}{1+\frac{1}{2}e^{-j\theta}}$

$H_1(e^{j0}) = \frac{2}{3}$, $H_1(e^{j\pi}) = 2$



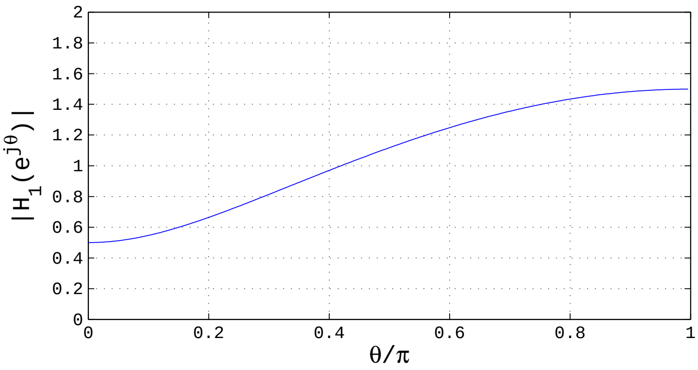
c) Mit Formelsammung: $h_1[n] = \left(-\dfrac{1}{2}\right)^n \sigma[n]$



d) System H_1 ist kausal ($h_1[n] = 0$, $n < 0$) und stabil ($\displaystyle\sum_{n=-\infty}^{\infty} |h_1[n]| < \infty$)

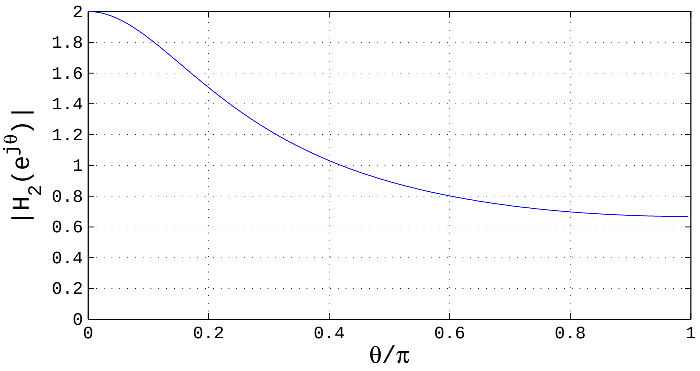
Gruppe B

a) $H_1(e^{j\theta}) = e^{-j\theta} - \frac{1}{2}e^{-j2\theta} = e^{-j\theta}\left(1 - \frac{1}{2}e^{-j\theta}\right)$, $\quad H_1(e^{j0}) = \frac{1}{2}$, $H_1(e^{j\pi}) = -\frac{3}{2}$

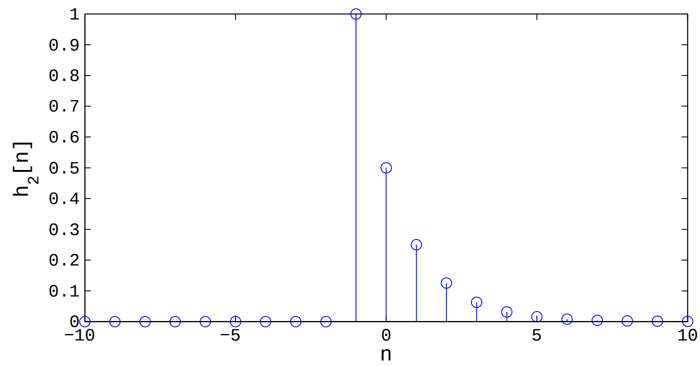


b) $H_1(e^{j\theta})H_2(e^{j\theta}) = 1, \rightarrow \; H_2(e^{j\theta}) = \dfrac{1}{H_1(e^{j\theta})} = \dfrac{e^{j\theta}}{1 - \frac{1}{2}e^{-j\theta}}$

$H_2(e^{j0}) = 2,\; H_2(e^{j\pi}) = -\frac{2}{3}$



c) Mit Formelsammung: $h_2[n] = \left(\dfrac{1}{2}\right)^{n+1} \sigma[n+1]$



d) System H_2 ist nicht kausal ($h_2[n] \neq 0,\; n < 0$), aber stabil ($\sum_{n=-\infty}^{\infty} |h_2[n]| < \infty$)