Name: ______________________________________

Matriculation number: ___________________________

Study code: ◯ 066 437 (master TC, old) ◯ 066 507 (master TC, new)
◯ 066 938 (master CE) ◯ 033 235 (bachelor EE)
◯ other: ____________

Instructions (READ CAREFULLY)

- This problem statement has to be handed in together with your solutions.
- A copy of the lecture notes is provided. You may use these lecture notes and a simple calculator during the exam. The use of any other documents or electronic devices is not permitted.
- Begin your solution to each problem on a new sheet of paper. If you do not attempt a problem, turn in a blank page on which you have written the missing problem number.
- Be sure to hand in your solutions by arranging them in such a way that the problems 1–4 appear in order.
- Check your results for plausibility. If you arrive at a result which is obviously incorrect, indicate that you are aware that the result is incorrect and elaborate, if possible.
- If the answer to one part depends upon the results of earlier parts that you were not able to answer, demonstrate your competence on the remaining parts by making reasonable assumptions about answers to the missing parts.
- When you are asked to make a sketch, do not forget to label the axes and mark significant points (e.g., maxima, minima, zeros, etc.).
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary.
- Note that the problems are weighted differently.
- The result of this exam will be sent to your generic student email address (e<matriculation_number>@student.tuwien.ac.at) within two weeks.

Problem	1	2	3	4	Sum
Max. credits	17	18	18	17	70
Credits					

Exercise credits: **Total credits:** **Grade (written part):**

Problem 1 (17 points) Consider two random variables x and y with joint probability density function

$$f_{\mathsf{x},\mathsf{y}}(x,y) = u(x)u(y)16e^{-4(x+y)},$$

where $u(\cdot)$ is the unit step function

$$u(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}.$$

Hint: $\displaystyle\int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$, for $a > 0$ and $n \in \mathbb{N} \cup \{0\}$.

a) Calculate the correlation $R_{\mathsf{x},\mathsf{y}}$ and covariance $C_{\mathsf{x},\mathsf{y}}$.

b) Are x and y statistically independent and/or uncorrelated and/or orthogonal? Justify your answer.

c) Consider the function

$$g(x,y) = \begin{cases} 5, & 0 < x \leq \frac{1}{2} \;\wedge\; 0 < y \leq \frac{1}{2} \\ -1, & \frac{1}{2} < x \;\vee\; \frac{1}{2} < y \\ 0, & \text{otherwise} \end{cases}.$$

c1) Sketch the function $g(x,y)$.

c2) Calculate the pmf of $\mathsf{z} = g(\mathsf{x},\mathsf{y})$.

c3) Calculate $\mathrm{E}\{\mathsf{z}\}$.

Problem 2 (18 points) Consider a random vector $\mathbf{v} = (\mathsf{v}_1 \;\; \mathsf{v}_2)^T$ with $\sigma^2_{\mathsf{v}_1} = \sigma^2_{\mathsf{v}_2} = 1.5$, $\mu_{\mathsf{v}_1} = \mu_{\mathsf{v}_2} = 2$, $R_{\mathsf{v}_1,\mathsf{v}_2} = \alpha$ and $R_{\mathsf{v}_2,\mathsf{v}_1} = \beta$.

a) Specify β in terms of α and find the possible value ranges for α and β.

b) For $\alpha = 3$, specify the correlation matrix $\boldsymbol{R}_{\mathbf{v}}$ and the covariance matrix $\boldsymbol{C}_{\mathbf{v}}$.

c) Find the eigenvalues λ_1 and λ_2 of $\boldsymbol{C}_{\mathbf{v}}$ for $\alpha = 3$.

d) Find the corresponding orthonormal eigenvectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$.

e) Find the whitening (decorrelation) transformation matrix $\boldsymbol{A}$.

f) Show that the resulting elements of the random vector $\mathbf{w} = \boldsymbol{A}\mathbf{v}$ are uncorrelated.

Problem 3 (18 points) Consider the real-valued random process $\mathsf{y}[n] = \mathsf{a}\sin(\theta_0 n) + \mathsf{v}[n]$, where $\mathsf{v}[n]$ is zero-mean, stationary and white with variance $\sigma_{\mathsf{v}}^2 = 1$ and a is a zero-mean random variable with variance σ_{a}^2 that is uncorrelated with the process $\mathsf{v}[n]$.

a) Calculate the mean of $\mathsf{y}[n]$.

b) Calculate the autocorrelation function and autocovariance function of $\mathsf{y}[n]$.

c) Is $\mathsf{y}[n]$ WSS and/or wide-sense cyclostationary? Justify your answer.

d) Consider the random vector

$$\mathbf{y} = \mathsf{a}\boldsymbol{w} + \mathbf{v},$$

where $\mathbf{y} = (\mathsf{y}[1], \ldots, \mathsf{y}[N])^T$, $\boldsymbol{w} = (\sin(\theta_0), \ldots, \sin(\theta_0 N))^T$ and $\mathbf{v} = (\mathsf{v}[1], \ldots, \mathsf{v}[N])^T$. Calculate the LMMSE estimator of a for given (observed) $\mathbf{y}$ and the resulting minimum mean square error.

Problem 4 (17 points) Consider two random variables x, y whose joint pdf $f_{\mathsf{x},\mathsf{y}}(x, y)$ is shown in Figure 1.

a) Provide a mathematical expression for $f_{\mathsf{x},\mathsf{y}}(x, y)$.

b) Provide expressions for the marginal pdf's $f_{\mathsf{x}}(x)$, $f_{\mathsf{y}}(y)$ and sketch them.

c) Calculate the probability $\mathrm{P}\{\mathsf{x} < -1, \mathsf{y} < 0\}$.

d) Are x und y statistically independent? Justify your answer.

e) Calculate the pdf of the random variable $\mathsf{z} = (\mathsf{x} + 1)^2$.

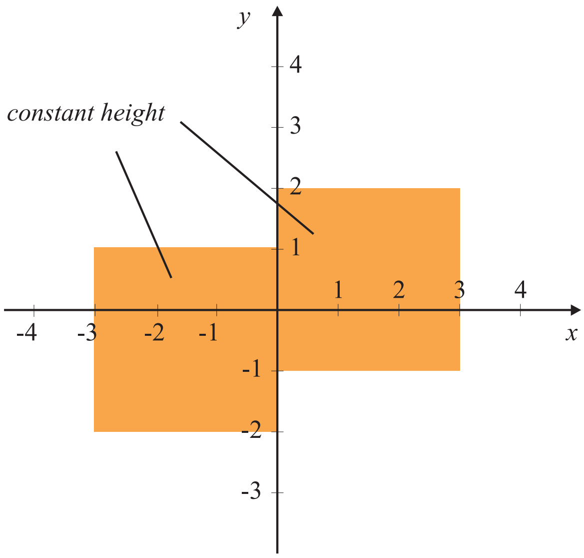

Figure 1: Joint pdf of x, y.