Signal Outage Optimized Beamforming for MISO TWDP Fading Channels

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Motivation

- Two-wave with diffuse power (TWDP) fading statistics:
  - Two dominant specular components with random relative phase
  - Diffuse background scattering
- TWDP fading has been observed in mmWave channel measurements
- Larger probability of deep fades than Rayleigh fading

⇒ Must be considered in reliable system design!
Contents

System Model

Outage-Optimized Beamforming

Numerical Results

Conclusions
System Model – MISO TWDP Fading

- Single-user input-output relationship
  \[ y = h^H f x + n \]

- Two-wave with diffuse power channel model
  \[ h = h_1 + h_2 + h_d, \]
  \[ h_d \sim \mathcal{N}_C(0, 2\sigma_d^2 I) \]

- Diffuse scattering component \( h_d \)
  - Specular components \( h_i = \sqrt{\gamma_i} e^{j\varphi_i} \bar{h}_i \)
  - Directional antenna array responses \( \bar{h}_1, \bar{h}_2 \), e.g.
    \[ \bar{h}_i = \frac{1}{\sqrt{N_t}} \left[ \ldots, e^{-j\frac{2\pi}{N_t}(\ell-1)\cos\theta_i}, \ldots \right]^H \]
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Beamforming over MISO TWDP Channels

- Effective beamformed channel

\[ z = re^{j\varphi} := h^H f = V_1 e^{j\psi_1} + V_2 e^{j\psi_2} + h_d^H f \]

- With unit-norm beamforming \( Z_d \sim \mathcal{N}_C (0, 2\sigma_d^2) \)

- Effective specular channel including the beamformer \( f \)

\[ V_i e^{j\psi_i} := h_i^H f, \]
\[ V_i = \sqrt{\gamma_i} \left| \bar{h}_i^H f \right|, \]
\[ \psi_i = \arg (\bar{h}_i^H f) - \varphi_i \sim \mathcal{U}(0, 2\pi) \]

- The amplitude \( r \) follows the TWDP fading model

\[ f_{TWDP}(r) = \frac{1}{2\pi} \int_0^{2\pi} f_{Rice}(r; K(1 + \Delta \cos \delta)) \, d\delta, \]
\[ K = \frac{V_1^2 + V_2^2}{2\sigma_d^2}, \quad \Delta = \frac{2V_1 V_2}{V_1^2 + V_2^2}, \quad \delta = \psi_1 - \psi_2 \sim \mathcal{U}(0, 2\pi) \]
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TWDP Fading Illustration

- Origin
- First specular component
- $V_1$
- Complex plane
TWDP Fading Illustration

- First specular component
- Second specular component

complex plane
TWDP Fading Illustration

Dependable Wireless Connectivity for the Society in Motion
TWDP Fading Illustration

Dependable Wireless Connectivity for the Society in Motion
TWDP Fading Illustration

PDF of diffuse component

V_1, V_2

V_1 - V_2

first specular component

V_1

PDF of specular envelope

V_1 + V_2

complex plane
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Outage-Optimized Beamforming

\[
\min_{f, \|f\|=1} P_{\text{out}}(f) = \min_{f, \|f\|=1} \mathbb{P}\{r \leq r_{\text{min}}\},
\]

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P_{\text{out}}(f) = \mathbb{P}\{r \leq r_{\text{min}}\} = \int_{0}^{r_{\text{min}}} f_{\text{TWDP}}(r) \, dr
\]

- Alternative: evaluation of the CDF requires a single integral of a Marcum Q-function
  \(\Rightarrow\) Obviously non-convex and global optimization is not feasible

- Considered approach: local gradient optimization

\[
\nabla_f P_{\text{out}} = \frac{1}{2\pi} \int_{0}^{r_{\text{min}}} \int_{0}^{2\pi} \nabla_f f_{\text{Rice}}(r; K(1 + \Delta \cos \delta)) \, d\delta \, dr
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- \(\nabla_f f_{\text{Rice}}(r; K(1 + \Delta \cos \delta))\) can be calculated in closed-form
- However, only numerical integration is possible
Outage-Optimized Beamforming

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\[
(1)
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Outage-Optimized Beamforming – Illustration

PDF of diffuse component

PDF of specular envelope

$V_1 - V_2$

$V_1 + V_2$

$r_{\text{min}}$

$V_2$

first specular component

second specular component

origin

diffuse component

specular component

envelope threshold

complex plane
Outage-Optimized Beamforming – Illustration

complex plane

PDF of diffuse component

PDF of specular envelope

$V_1 - V_2$

$V_1 + V_2$

$r_{\text{min}}$

origin

envelope threshold

PDF of diffuse component

PDF of specular envelope

$V_1 - V_2$

$V_1 + V_2$

$r_{\text{min}}$

origin

envelope threshold

complex plane
Outage-Optimized Beamforming – Upper Bound and Approximation

PDF of diffuse component

PDF of specular component

$V_1 - V_2$

$V_1 + V_2$

$r_{min}$

origin

complex plane

$P_{out}(f) \leq \left( Q \left( \frac{-r_{min}}{\sigma_d} \right) - Q \left( \frac{r_{min}}{\sigma_d} \right) \right) \cdot \int_{|V_1 - V_2|}^{V_1 + V_2} \left( Q \left( \frac{r - r_{min}}{\sigma_d} \right) - Q \left( \frac{r + r_{min}}{\sigma_d} \right) \right) f_{TW}(r) dr$
Outage-Optimized Beamforming – Upper Bound and Approximation

PDF of diffuse component

PDF of specular component

$P_{\text{out}}(f) \approx \frac{1}{4V_2} \left( \sqrt{\frac{2\sigma_d^2}{\pi}} \left( -e^{-\frac{a_1^2}{2\sigma_d^2}} + e^{-\frac{a_2^2}{2\sigma_d^2}} - e^{-\frac{a_3^2}{2\sigma_d^2}} + e^{-\frac{a_4^2}{2\sigma_d^2}} \right) \right.
\left. - a_1 \text{erf} \left( \frac{a_1}{\sqrt{2\sigma_d^2}} \right) + a_2 \text{erf} \left( \frac{a_2}{\sqrt{2\sigma_d^2}} \right) - a_3 \text{erf} \left( \frac{a_3}{\sqrt{2\sigma_d^2}} \right) + a_4 \text{erf} \left( \frac{a_4}{\sqrt{2\sigma_d^2}} \right) \right)$
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Outage-Probability, Upper Bound and Approximation

- Equally strong specular components $V_1^2 = V_2^2 = 1$ and diffuse component $\sigma_d^2 = 1$
Outage-Optimized Beamforming – Approximation vs. Accurate Gradient

- $N_t = 4$ transmit antennas, equal macroscopic pathloss $\gamma_1 = \gamma_2 = 2$
- Averaging over random angles $\theta_i$, phases $\varphi_i$ and diffuse channels $h_d$
Outage-Optimized Beamforming – Comparison to MRT and ZF

- Equal and unequal macroscopic pathloss, $\sigma_d^2 = 0.1$
- MRT over stronger specular component, ZF of weaker specular component
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Summary:

- TWDP fading generalizes Rician and Rayleigh fading
- It has been observed in several mmWave measurement campaigns
- It potentially causes fading that is even worse than Rayleigh fading!
- Outage-optimal beamforming for TWDP fading is complicated

Future Work:

- Good heuristic beamformers with low computational complexity
- Multi-user beamforming under TWDP fading
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- Rician vs. two-wave with diffuse power (TWDP) fading statistics
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